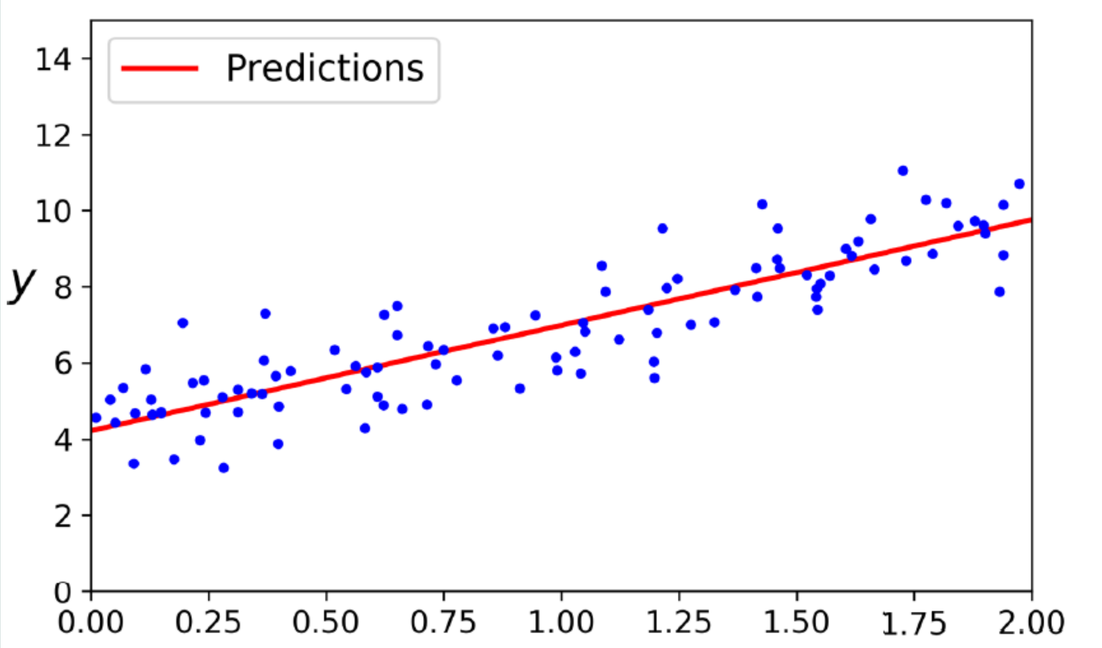
Mustafa Shiple

# Linear Regression

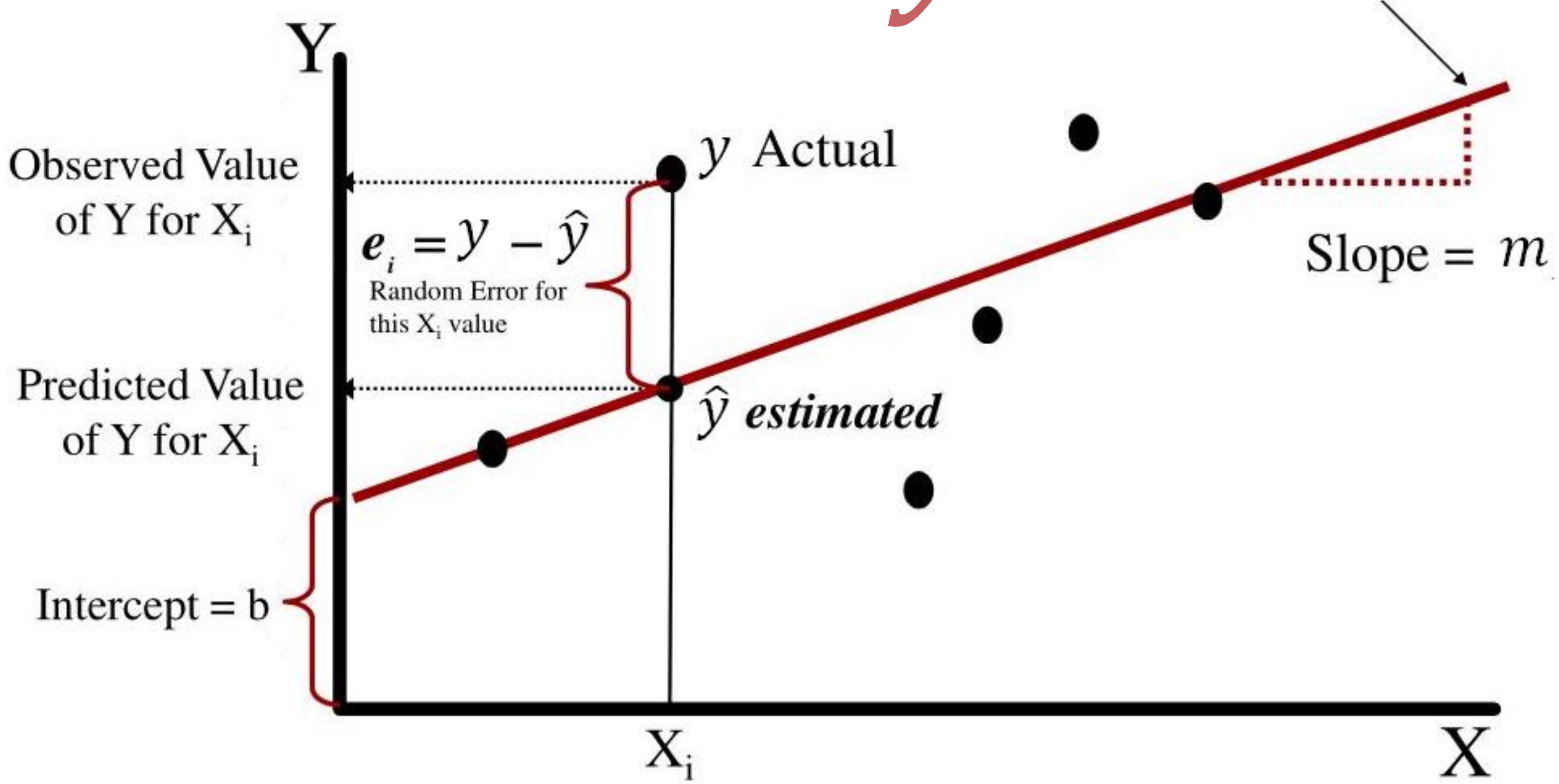
# Introduction

- "fitting a straight line"
- a statistical analysis method to determine the quantitative relationships between two or more variables through regression analysis in mathematical statistics.



# Univariate linear regression (Simple LR)

$$\hat{y} = mx + b$$

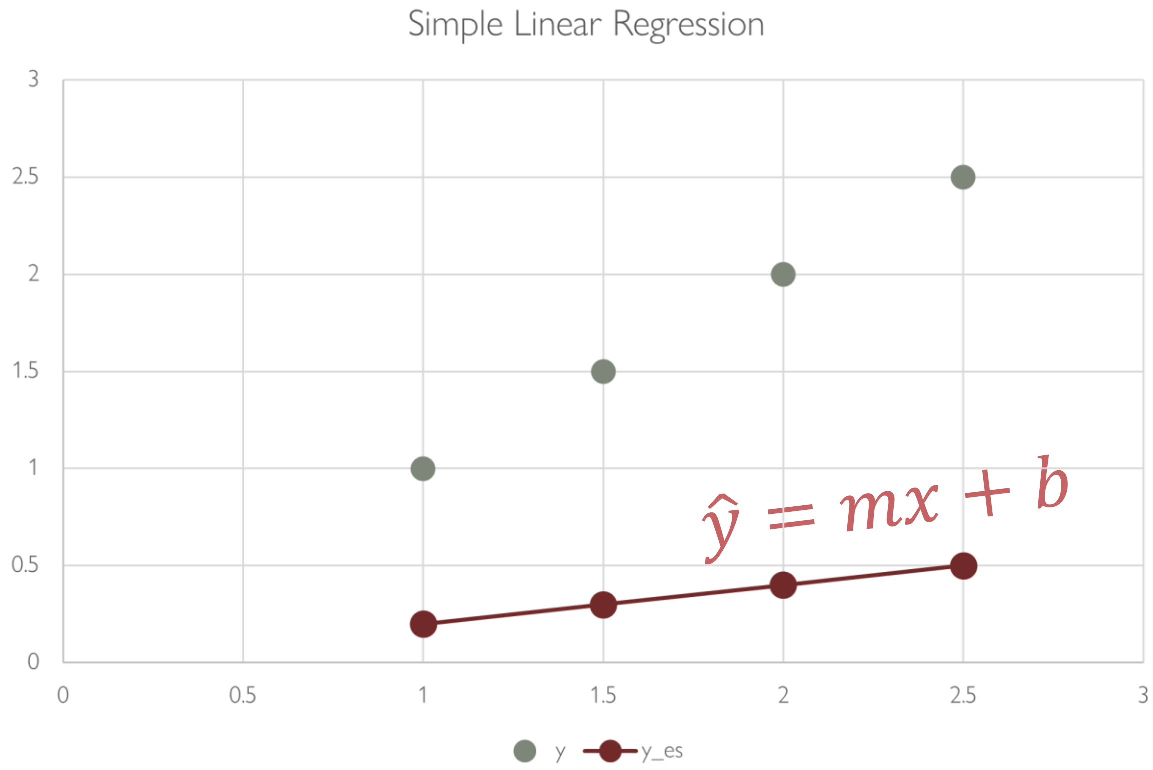


# loss function (**Mean Squared Error (MSE)**)

$$J(w) = \frac{1}{2n}\sum_{i=1}^{n}\left(y^{(i)} - \hat{y}^{(i)}\right)^2$$


$$J(w) = \frac{1}{2*4}\sum_{i=1}^{4}\{\,(1-0.2)^2, (1.5-0.3)^2, (2-0.4)^2, (2.5-0.5)^2\}$$

| base | slop | | |
|---|---|---|---|
| 0 | 0.2 | | |
| x | y | y_es | square error |
| 1 | 1 | 0.2 | 0.64 |
| 1.5 | 1.5 | 0.3 | 1.44 |
| 2 | 2 | 0.4 | 2.56 |
| 2.5 | 2.5 | 0.5 | 4 |
| mean square error | | | 1.08 |



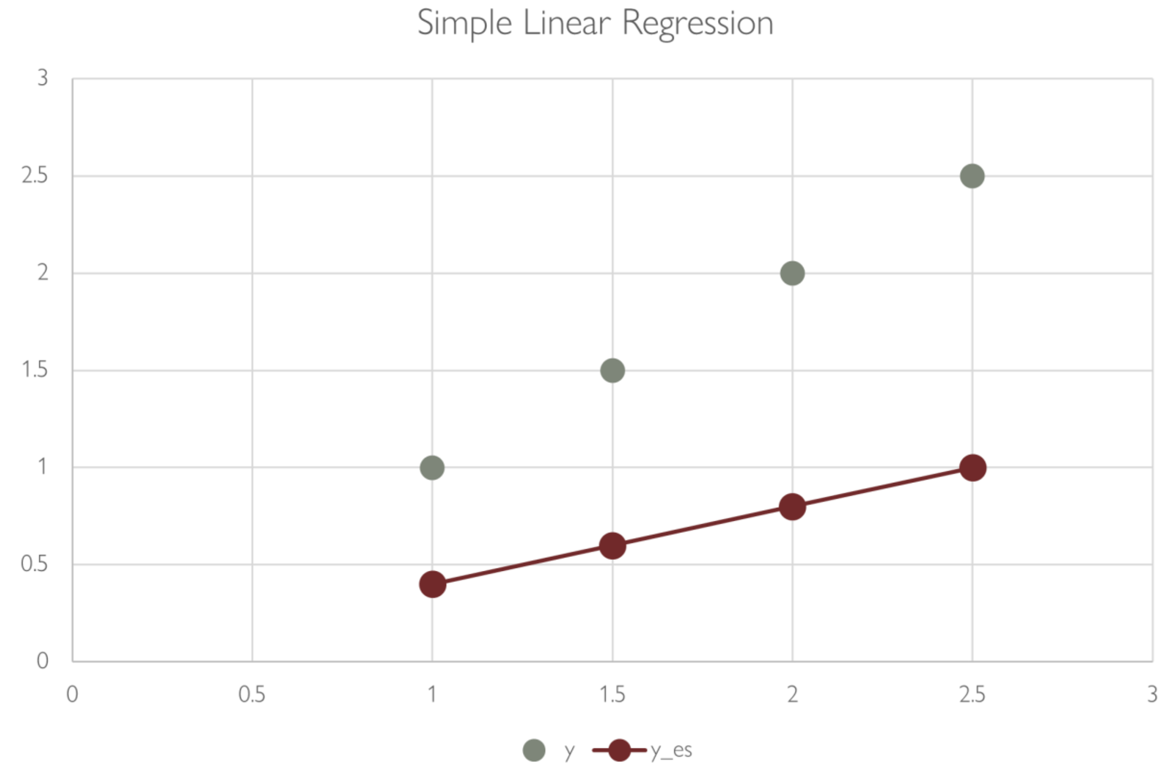
# squared-error loss function

$$J(w) = \frac{1}{2n}\sum_{i=1}^{n}(y^{(i)} - \hat{y}^{(i)})^2$$

0.61

$$J(w) = \frac{1}{2 * 4}\sum_{i=1}^{4}\{ (1 - 0.4)^2, (1.5 - 0.6)^2, (2 - 0.8)^2, (2.5 - 1)^2\}$$

| base | slop | | | |
|---|---|---|---|---|
| 0 | 0.4 | | | |
| x | y | y_es | square error | square error |
| 1 | 1 | 0.4 | 0.36 | 0.64 |
| 1.5 | 1.5 | 0.6 | 0.81 | 1.44 |
| 2 | 2 | 0.8 | 1.44 | 2.56 |
| 2.5 | 2.5 | 1 | 2.25 | 4 |
| mean square error | | | 0.61 | 1.08 |

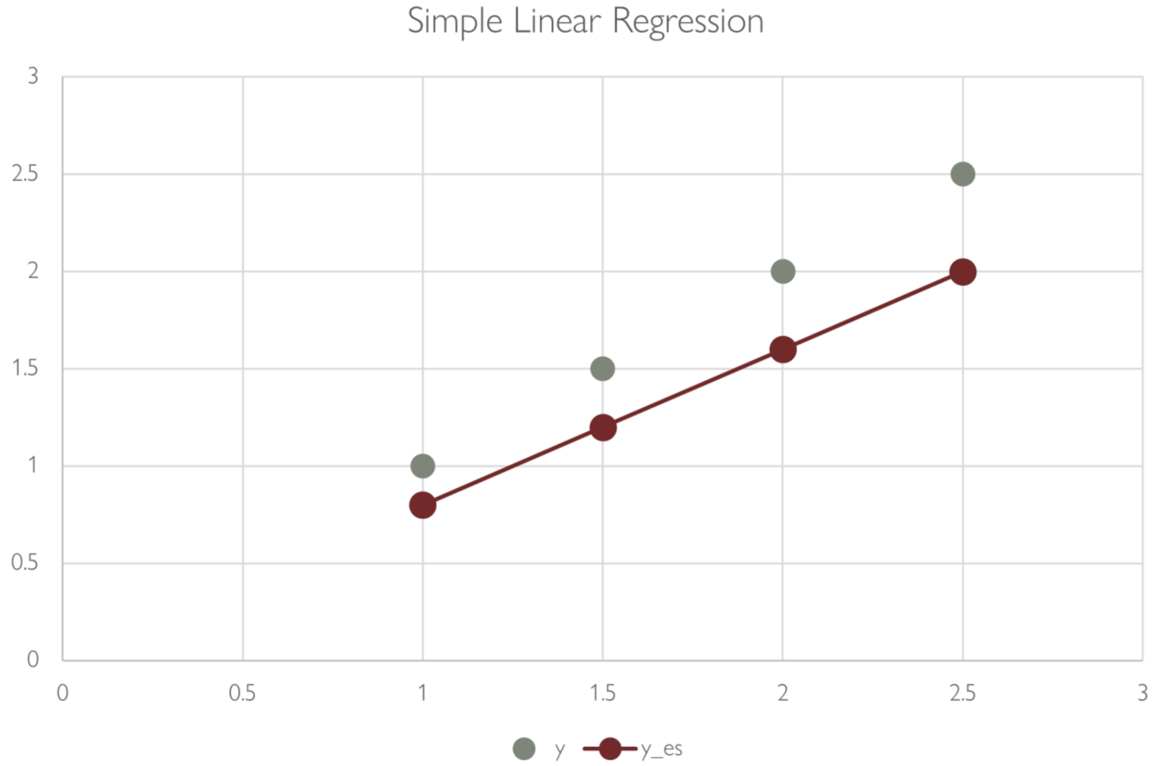


# loss function

$$J(w) = \frac{1}{2n}\sum_{i=1}^{n}(y^{(i)} - \hat{y}^{(i)})^2$$


$$J(w) = \frac{1}{2 * 4}\sum_{i=1}^{4}\{\,(1-0.8)^2, (1.5-1.2)^2\\ ,(2-1.6)^2, (2.5-2)^2\}$$

| base | slop | | | | |
|---|---|---|---|---|---|
| 0 | 0.8 | | | | |
| x | y | y_es | square error | square error | square error |
| 1 | 1 | 0.8 | 0.04 | 0.36 | 0.64 |
| 1.5 | 1.5 | 1.2 | 0.09 | 0.81 | 1.44 |
| 2 | 2 | 1.6 | 0.16 | 1.44 | 2.56 |
| 2.5 | 2.5 | 2 | 0.25 | 2.25 | 4 |
| mean square error | | | 0.07 | 0.61 | 1.08 |

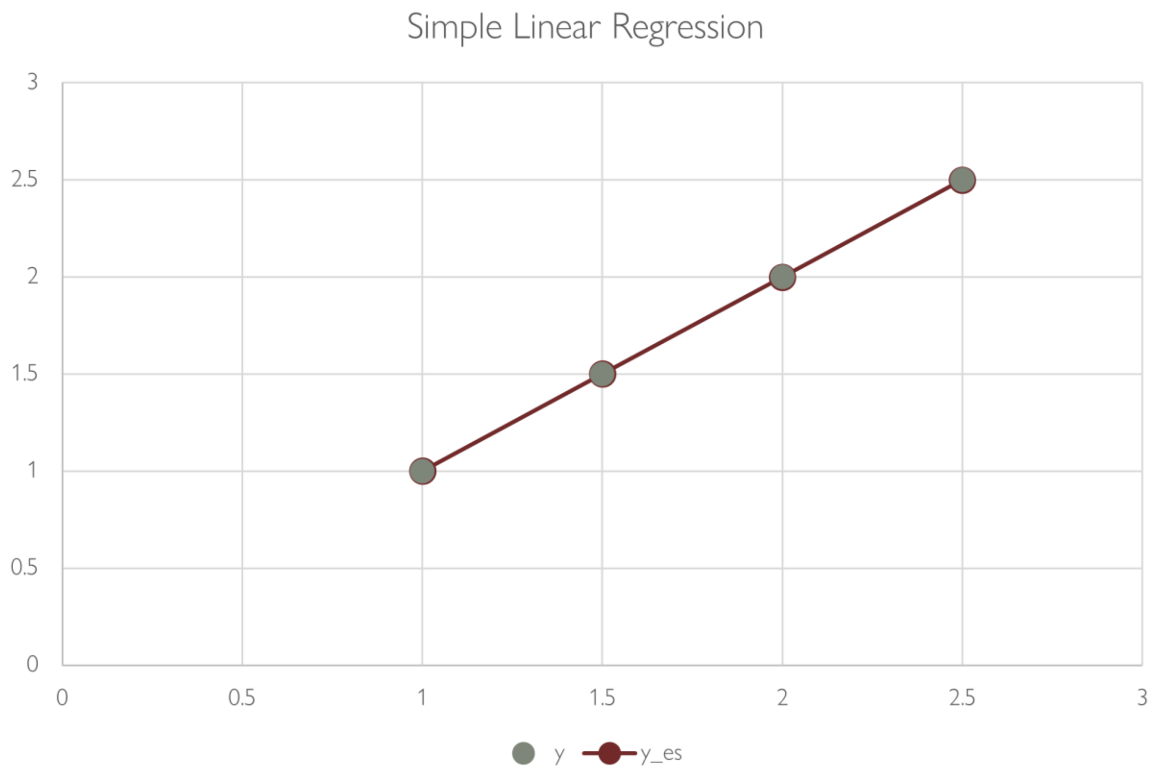


# loss function

$$J(w) = \frac{1}{2n}\sum_{i=1}^{n}(y^{(i)} - \hat{y}^{(i)})^2$$


$$J(w) = \frac{1}{2*4}\sum_{i=1}^{4}\{(1-1)^2, (1.5-1.5)^2, (2-2)^2, (2.5-2.5)^2\}$$

| base | slop | | | | | |
|---|---|---|---|---|---|---|
| 0 | 1 | | | | | |
| x | y | y_es | square error | square error | square error | square error |
| 1 | 1 | 1 | 0 | 0.04 | 0.36 | 0.64 |
| 1.5 | 1.5 | 1.5 | 0 | 0.09 | 0.81 | 1.44 |
| 2 | 2 | 2 | 0 | 0.16 | 1.44 | 2.56 |
| 2.5 | 2.5 | 2.5 | 0 | 0.25 | 2.25 | 4 |
| mean square error | | | 0 | 0.07 | 0.61 | 1.08 |

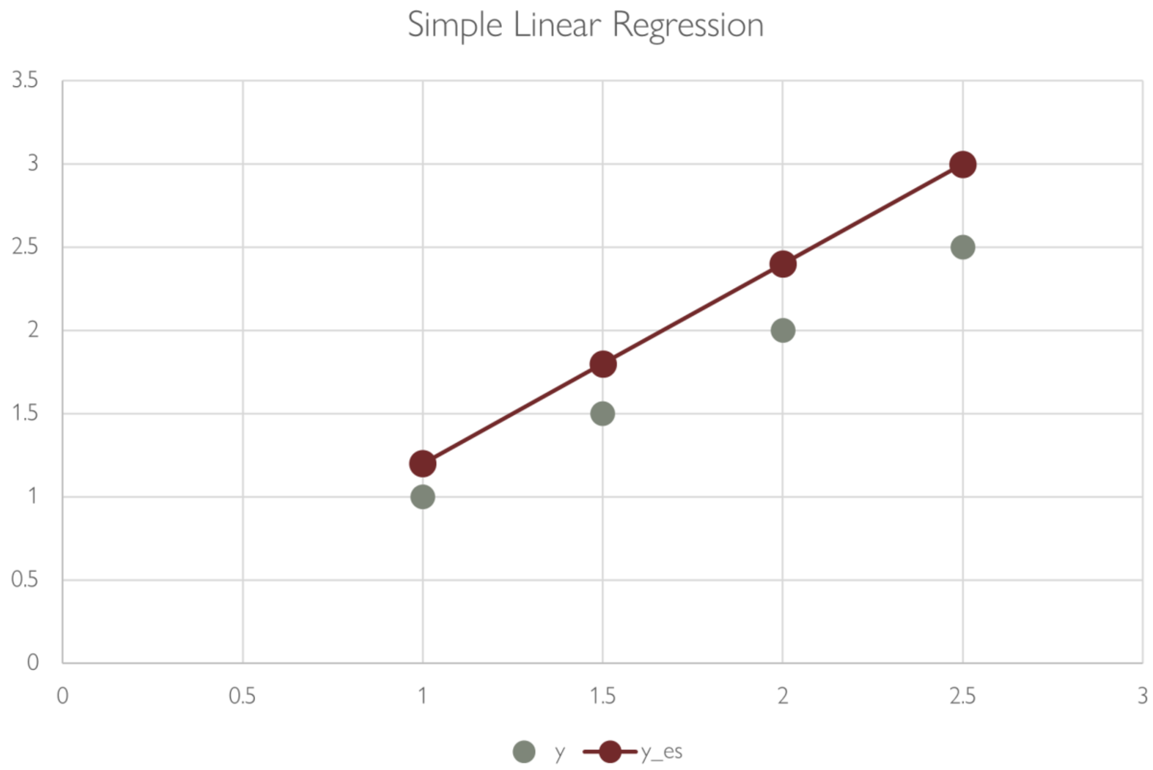


# squared-error loss function

$$J(w) = \frac{1}{2n}\sum_{i=1}^{n}(y^{(i)} - \hat{y}^{(i)})^2$$


$$J(w) = \frac{1}{2*4}\sum_{i=1}^{4}\{\,(1-1.2)^2, (1.5-1.8)^2, (2-2.4)^2, (2.5-3)^2\}$$

| base | slop | | | | | | |
|---|---|---|---|---|---|---|---|
| 0 | 1.2 | | | | | | |
| x | y | y_es | square error | square error | square error | square error | square error |
| 1 | 1 | 1.2 | 0.04 | 0 | 0.04 | 0.36 | 0.64 |
| 1.5 | 1.5 | 1.8 | 0.09 | 0 | 0.09 | 0.81 | 1.44 |
| 2 | 2 | 2.4 | 0.16 | 0 | 0.16 | 1.44 | 2.56 |
| 2.5 | 2.5 | 3 | 0.25 | 0 | 0.25 | 2.25 | 4 |
| mean square error | | | 0.07 | 0 | 0.07 | 0.61 | 1.08 |



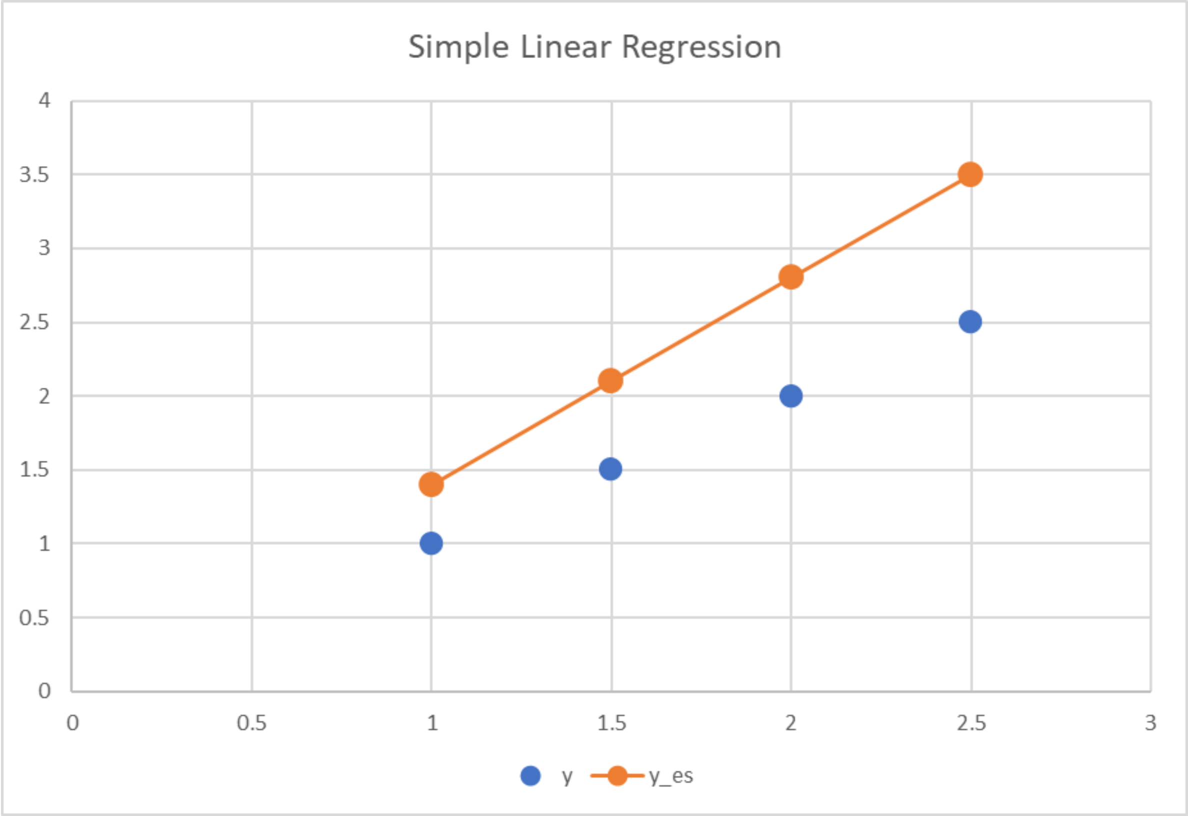
# loss function

$$J(w) = \frac{1}{2n}\sum_{i=1}^{n}(y^{(i)} - \hat{y}^{(i)})^2$$

# 0.27

$$J(w) = \frac{1}{2*4}\sum_{i=1}^{4}\{(1-1.4)^2, (1.5-2.1)^2, (2-2.8)^2, (2.5-3.5)^2\}$$

| base | slop | | | | | | | |
|---|---|---|---|---|---|---|---|---|
| 0 | 1.4 | | | | | | | |
| x | y | y_es | square error | square error | square error | square error | square error | square error |
| 1 | 1 | 1.4 | 0.16 | 0.04 | 0 | 0.04 | 0.36 | 0.64 |
| 1.5 | 1.5 | 2.1 | 0.36 | 0.09 | 0 | 0.09 | 0.81 | 1.44 |
| 2 | 2 | 2.8 | 0.64 | 0.16 | 0 | 0.16 | 1.44 | 2.56 |
| 2.5 | 2.5 | 3.5 | 1 | 0.25 | 0 | 0.25 | 2.25 | 4 |
| mean square error | | | 0.27 | 0.07 | 0 | 0.07 | 0.61 | 1.08 |

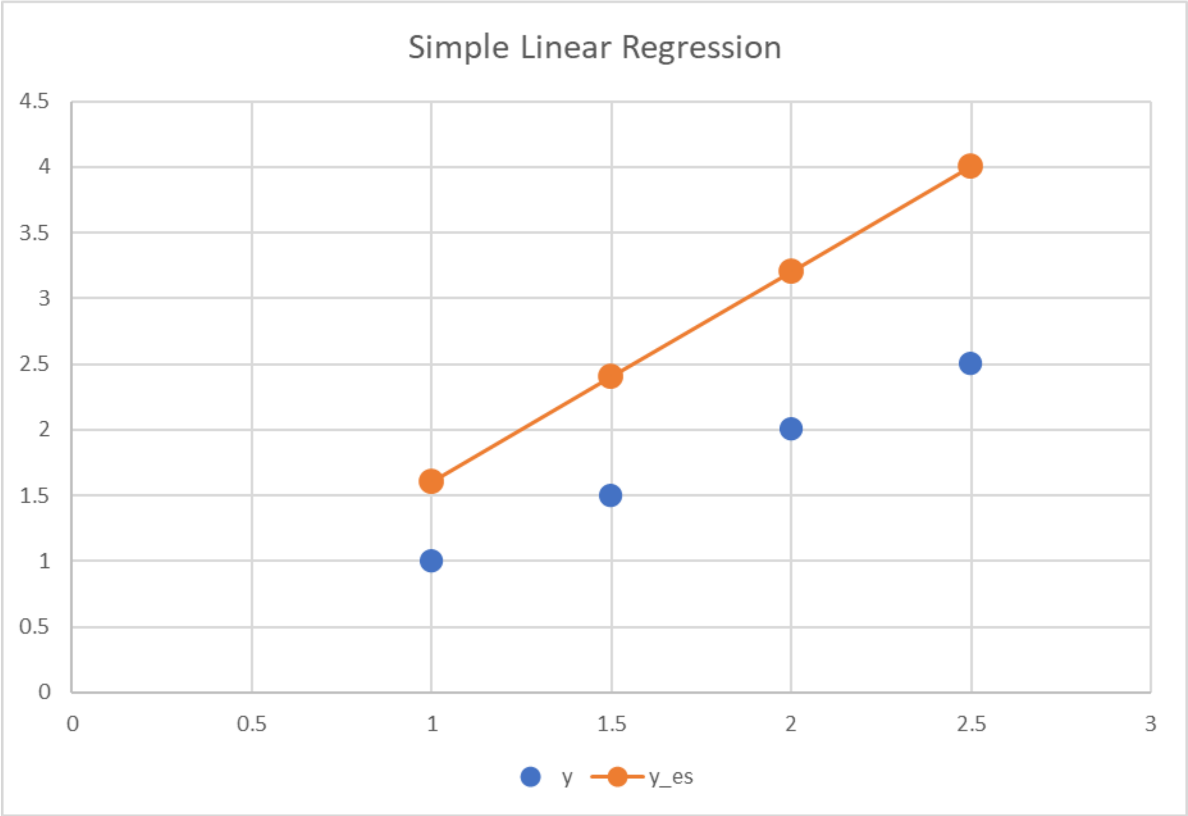


# loss function

$$J(w) = \frac{1}{2n}\sum_{i=1}^{n}(y^{(i)} - \hat{y}^{(i)})^2$$

| base | slop | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 1.6 | | | | | | | | |
| x | y | y_es | square error | square error | square error | square error | square error | square error | square error |
| 1 | 1 | 1.6 | 0.36 | 0.16 | 0.04 | 0 | 0.04 | 0.36 | 0.64 |
| 1.5 | 1.5 | 2.4 | 0.81 | 0.36 | 0.09 | 0 | 0.09 | 0.81 | 1.44 |
| 2 | 2 | 3.2 | 1.44 | 0.64 | 0.16 | 0 | 0.16 | 1.44 | 2.56 |
| 2.5 | 2.5 | 4 | 2.25 | 1 | 0.25 | 0 | 0.25 | 2.25 | 4 |
| mean square error | | | 0.61 | 0.27 | 0.07 | 0 | 0.07 | 0.61 | 1.08 |

0.61



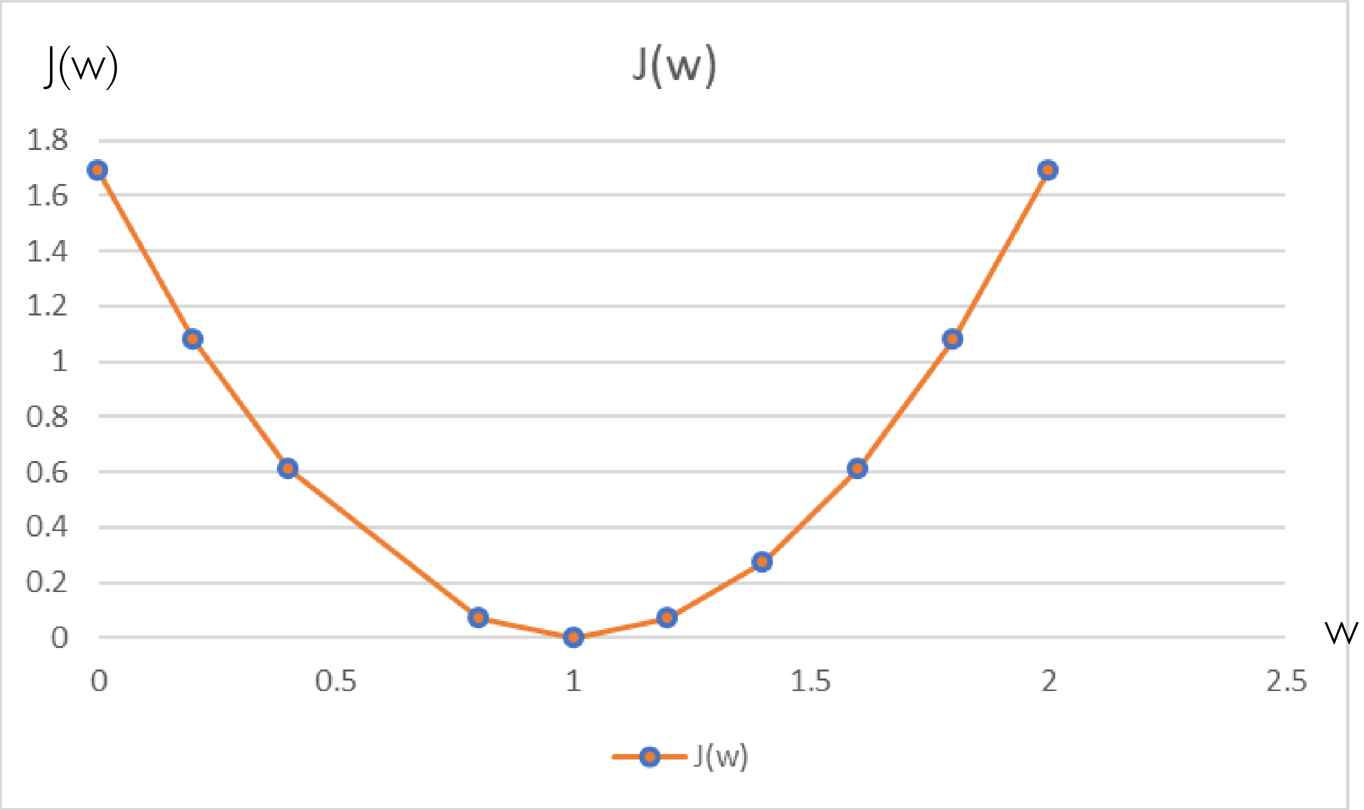

$$J(w) = \frac{1}{2 * 4}\sum_{i=1}^{4}\{ (1 - 1.6)^2, (1.5 - 2.4)^2, (2 - 3.2)^2, (2.5 - 4)^2\}$$

# loss function

$$J(w) = \frac{1}{2n}\sum_{i=1}^{n}(y^{(i)} - \hat{y}^{(i)})^2$$

Cost function

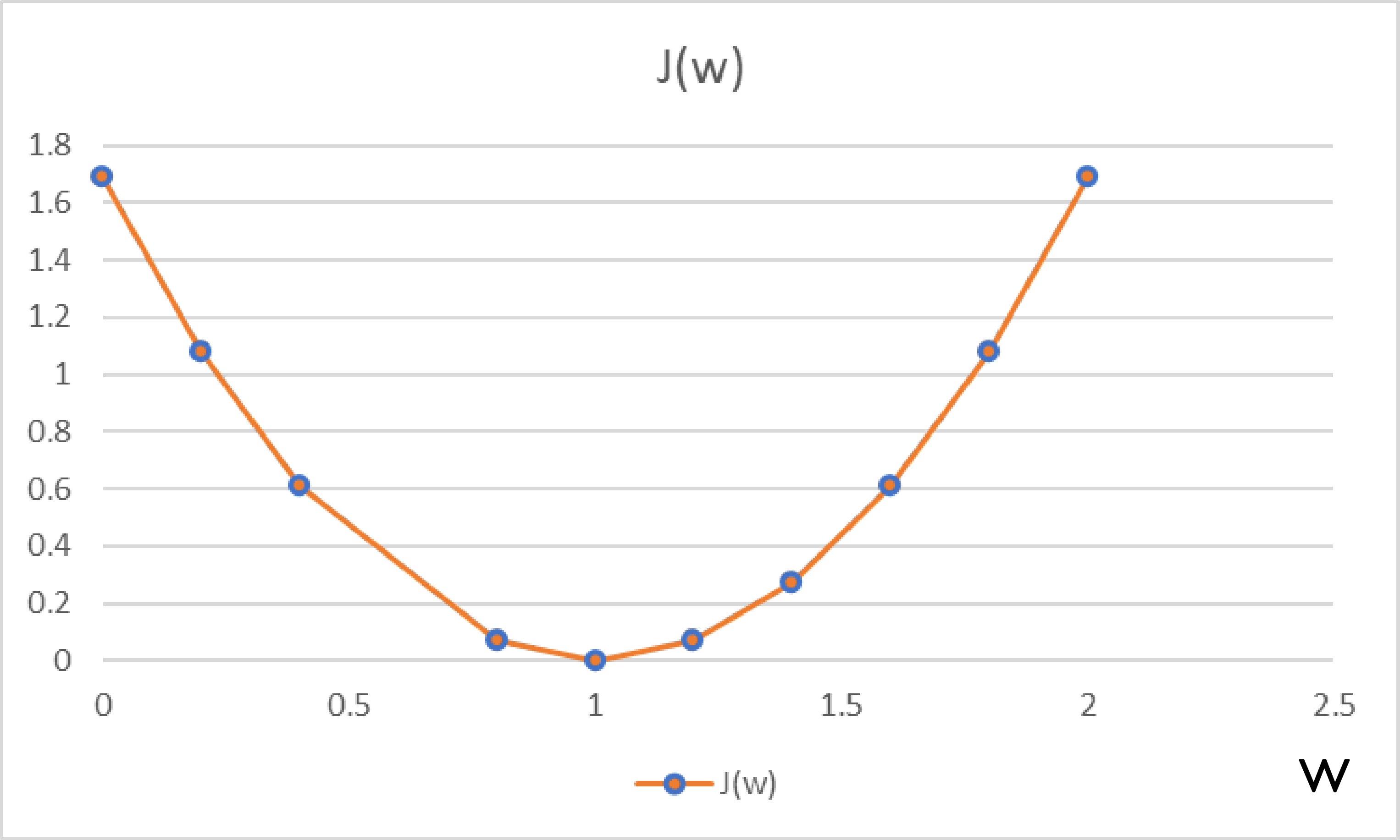
| base | slop | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 1.6 | | | | | | | | |
| x | y | y_es | square error | square error | square error | square error | square error | square error | square error |
| 1 | 1 | 1.6 | 0.36 | 0.16 | 0.04 | 0 | 0.04 | 0.36 | 0.64 |
| 1.5 | 1.5 | 2.4 | 0.81 | 0.36 | 0.09 | 0 | 0.09 | 0.81 | 1.44 |
| 2 | 2 | 3.2 | 1.44 | 0.64 | 0.16 | 0 | 0.16 | 1.44 | 2.56 |
| 2.5 | 2.5 | 4 | 2.25 | 1 | 0.25 | 0 | 0.25 | 2.25 | 4 |
| mean square error | | | 0.61 | 0.27 | 0.07 | 0 | 0.07 | 0.61 | 1.08 |



# Gradient Descent

1. Keep changing w,b to reduce J(w, b)
2. Until we settle at or near a minimum

| base | slop | | |
|---|---|---|---|
| 0 | 1.6 | | |
| x | y | y_es | square error |
| 1 | 1 | 1.6 | 0.36 |
| 1.5 | 1.5 | 2.4 | 0.81 |
| 2 | 2 | 3.2 | 1.44 |
| 2.5 | 2.5 | 4 | 2.25 |
| mean square error | | | 0.61 |



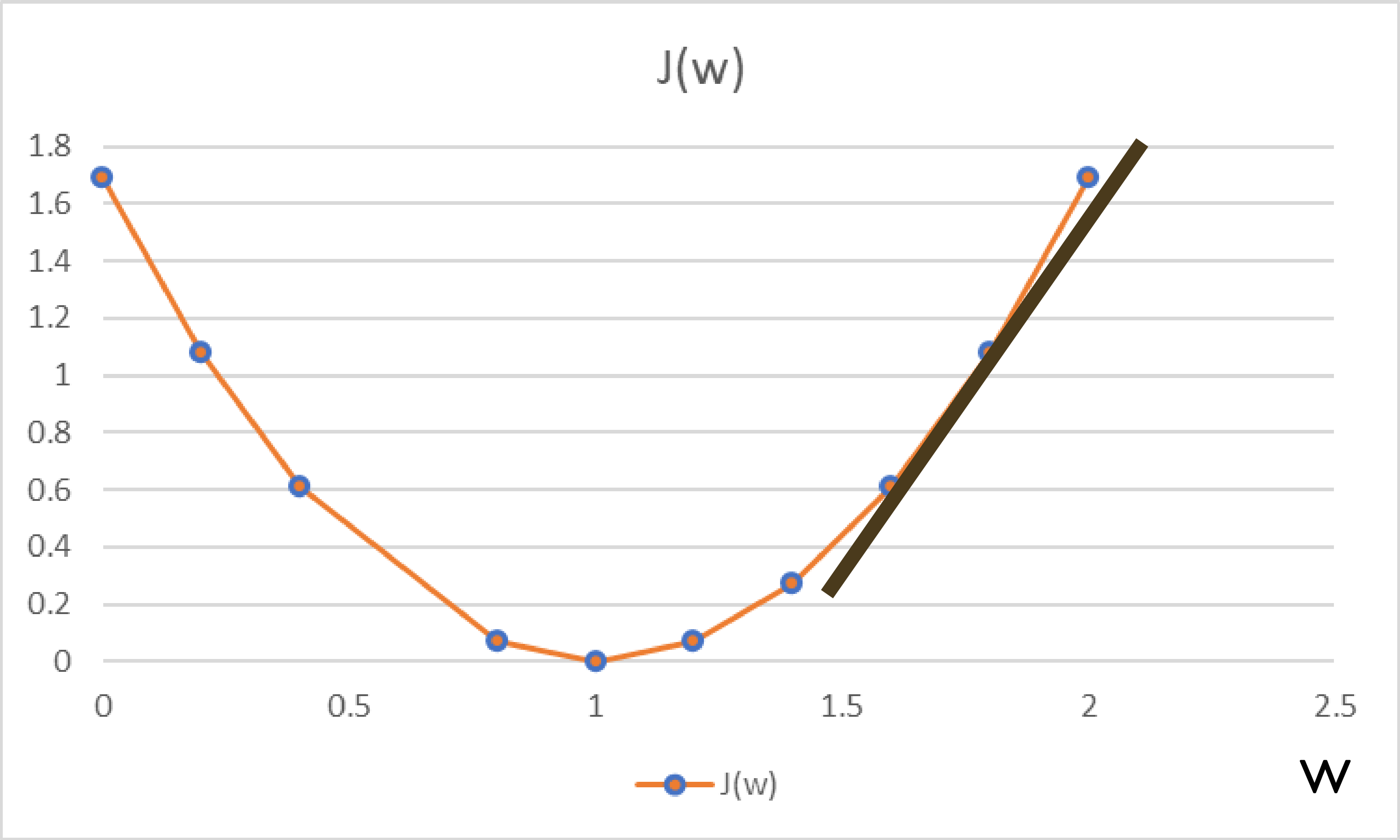

$$w = w - \alpha \frac{\partial}{\partial w} J(w, b)$$

Learning Rate (step)

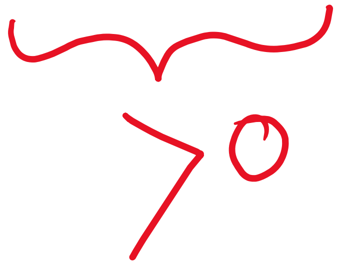
$$b = b - \alpha \frac{\partial}{\partial b} J(w, b)$$

1. Keep changing w,b to reduce J(w, b)
2. Until we settle at or near a minimum

| base | slop | | |
|---|---|---|---|
| 0 | 1.6 | | |
| x | y | y_es | square error |
| 1 | 1 | 1.6 | 0.36 |
| 1.5 | 1.5 | 2.4 | 0.81 |
| 2 | 2 | 3.2 | 1.44 |
| 2.5 | 2.5 | 4 | 2.25 |
| mean square error | | | 0.61 |

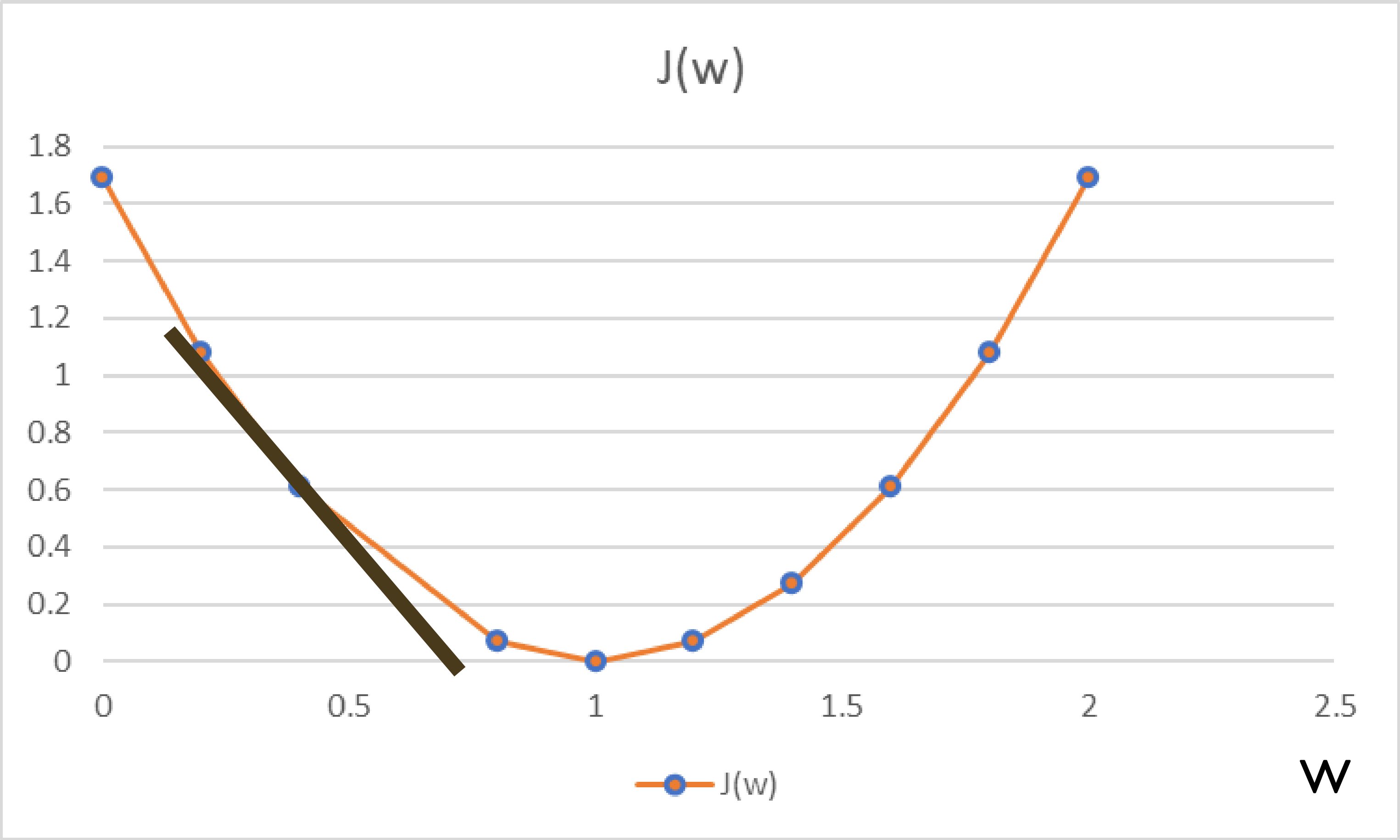



$$w = w - \alpha \frac{\partial}{\partial w} J(w, b)$$



1. Keep changing w,b to reduce J(w, b)
2. Until we settle at or near a minimum

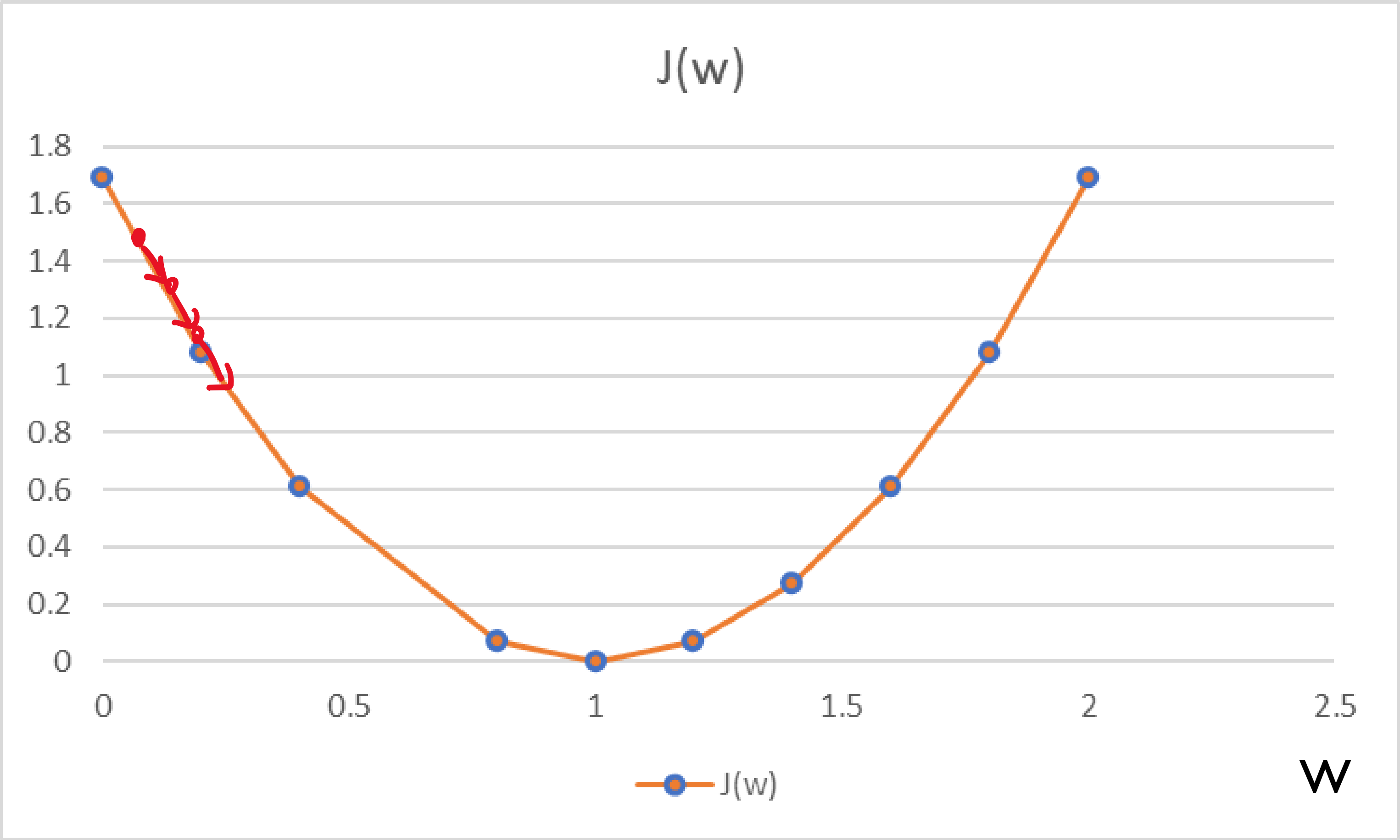
| base | slop | | |
|---|---|---|---|
| 0 | 1.6 | | |
| x | y | y_es | square error |
| 1 | 1 | 1.6 | 0.36 |
| 1.5 | 1.5 | 2.4 | 0.81 |
| 2 | 2 | 3.2 | 1.44 |
| 2.5 | 2.5 | 4 | 2.25 |
| mean square error | | | 0.61 |




$$w = w - \alpha \frac{\partial}{\partial w} J(w, b)$$

$< 0$

# 1. Learning rate

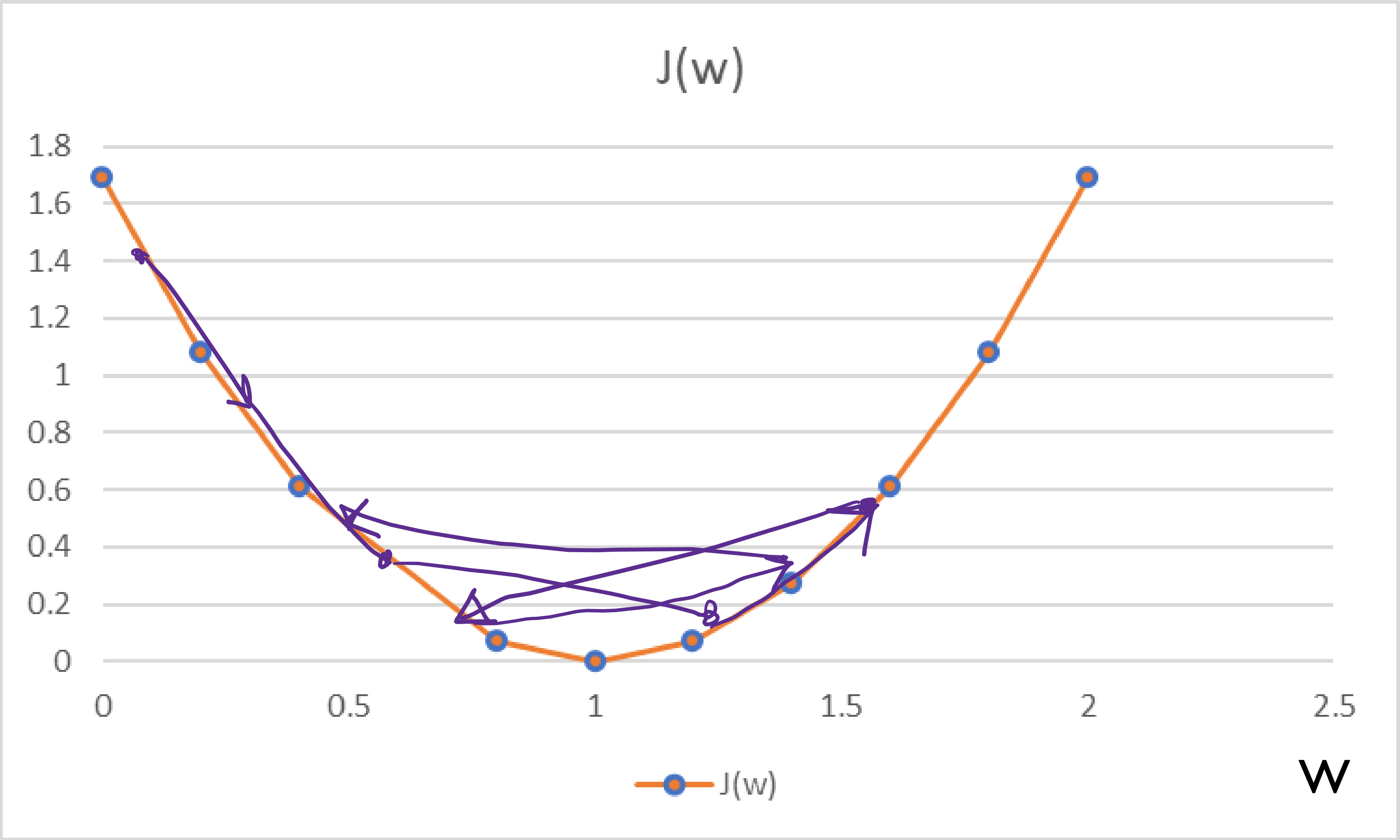



$$w = w - \alpha \frac{\partial}{\partial w} J(w, b)$$

α <<

too many steps

# 1. Learning rate

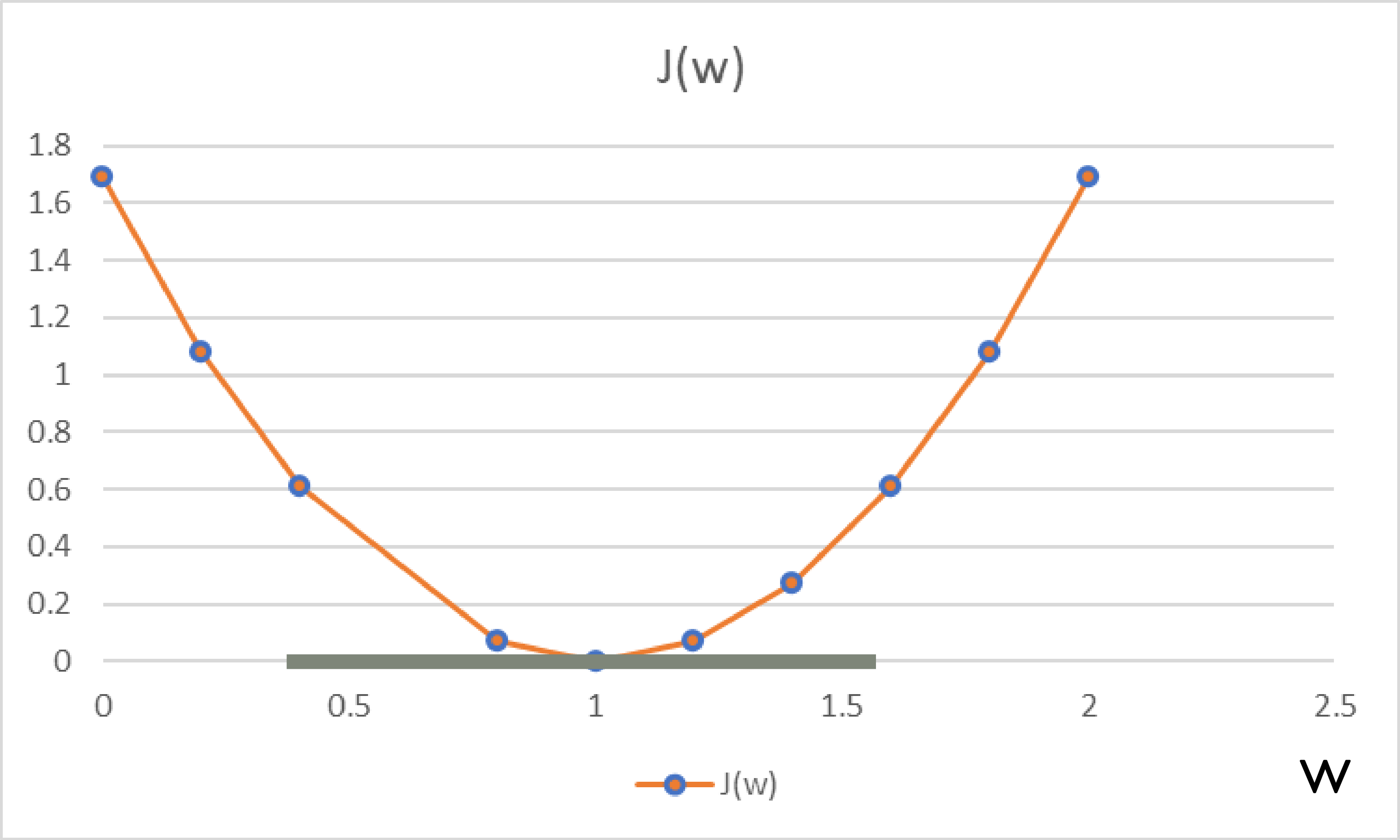



$$w = w - \alpha \frac{\partial}{\partial w} J(w, b)$$

a larger value

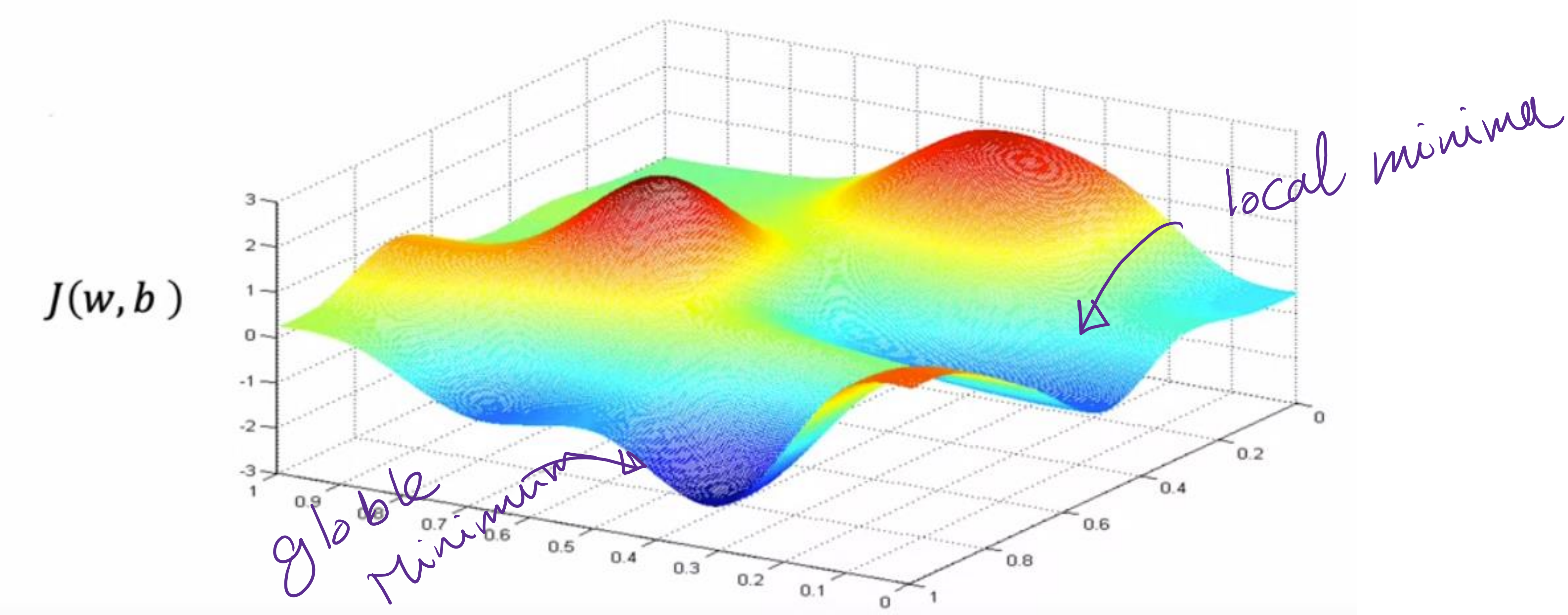
1. Keep changing w,b to reduce J(w, b)
2. Until we settle at or near a minimum

| base | slop | | |
|---|---|---|---|
| 0 | 1.6 | | |
| x | y | y_es | square error |
| 1 | 1 | 1.6 | 0.36 |
| 1.5 | 1.5 | 2.4 | 0.81 |
| 2 | 2 | 3.2 | 1.44 |
| 2.5 | 2.5 | 4 | 2.25 |
| mean square error | | | 0.61 |



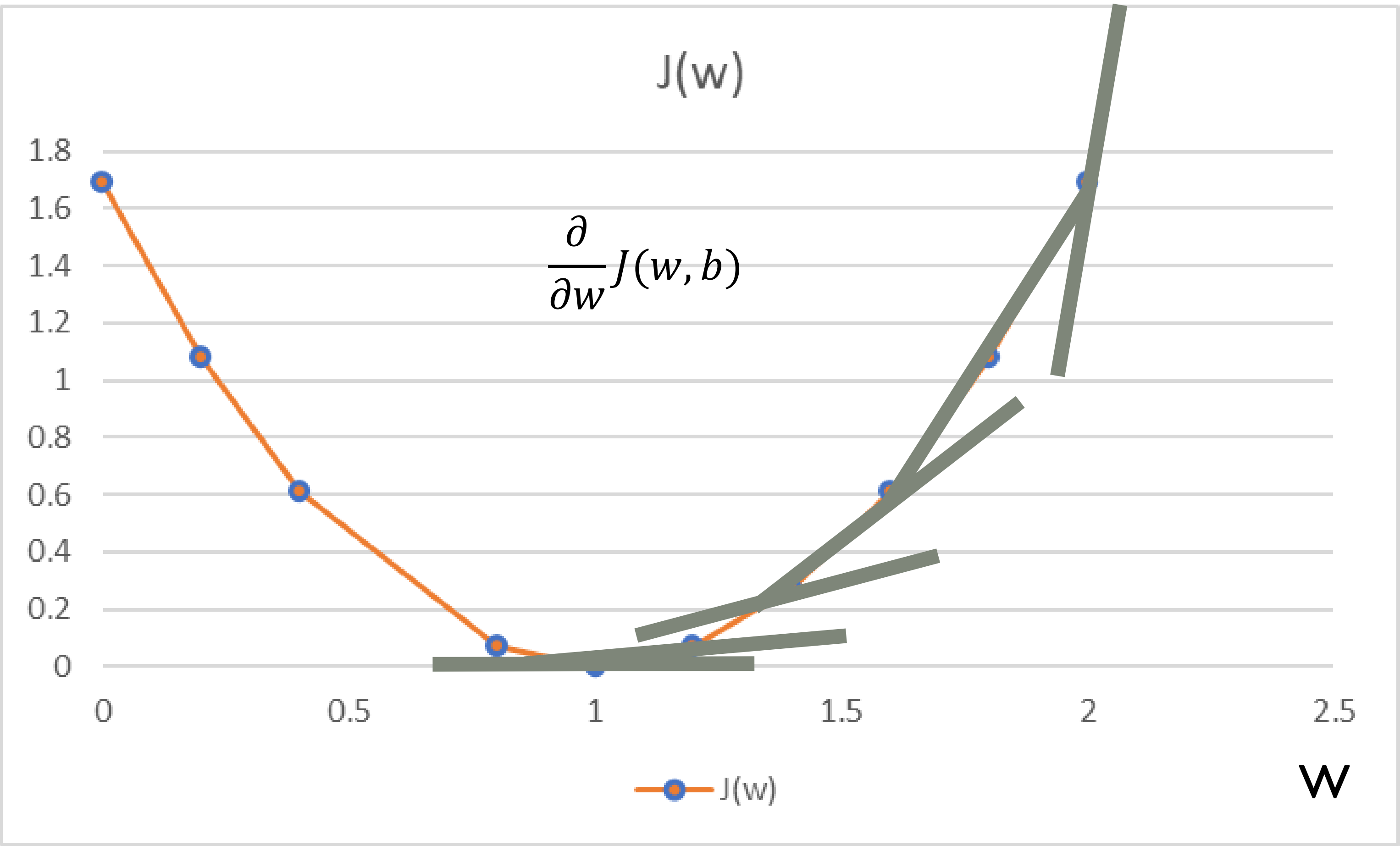

$$w = w - \alpha \times 0$$
$$w = w$$

# Local minima



1. Keep changing w,b to reduce J(w, b)
2. Until we settle at or near a minimum

| base | slop | | |
|---|---|---|---|
| 0 | 1.6 | | |
| x | y | y_es | square error |
| 1 | 1 | 1.6 | 0.36 |
| 1.5 | 1.5 | 2.4 | 0.81 |
| 2 | 2 | 3.2 | 1.44 |
| 2.5 | 2.5 | 4 | 2.25 |
| mean square error | | | 0.61 |




$$\frac{\partial}{\partial w}J(w,b)$$

$$w = w - \alpha \frac{\partial}{\partial w}J(w,b)$$

$\frac{\partial}{\partial w}J(w,b)$ ↓ when go further

# Gradient Descent

$$w = w - \alpha \frac{\partial}{\partial w} J(w, b)$$

$$\frac{\partial}{\partial w} J(w, b) = \frac{\partial}{\partial w} \left[ \frac{1}{2n} \sum_{i=1}^{n} ((wx + b) - \hat{y}^{(i)})^2 \right]$$

$$= \frac{1}{2n} \times 2\left[\sum_{i=1}^{n} ((wx^{(i)} + b) - \hat{y}^{(i)})) \times x^{(i)}\right]$$

$$= \frac{1}{n} \times \sum_{i=1}^{n} (y - \hat{y}^{(i)}) \times x^{(i)}$$

# Gradient Descent

$$b = b - \alpha \frac{\partial}{\partial b} J(w, b)$$

$$\frac{\partial}{\partial b} J(w, b) = \frac{\partial}{\partial b}\left[\frac{1}{2n}\sum_{i=1}^{n}((wx + b) - \hat{y}^{(i)})^2\right]$$

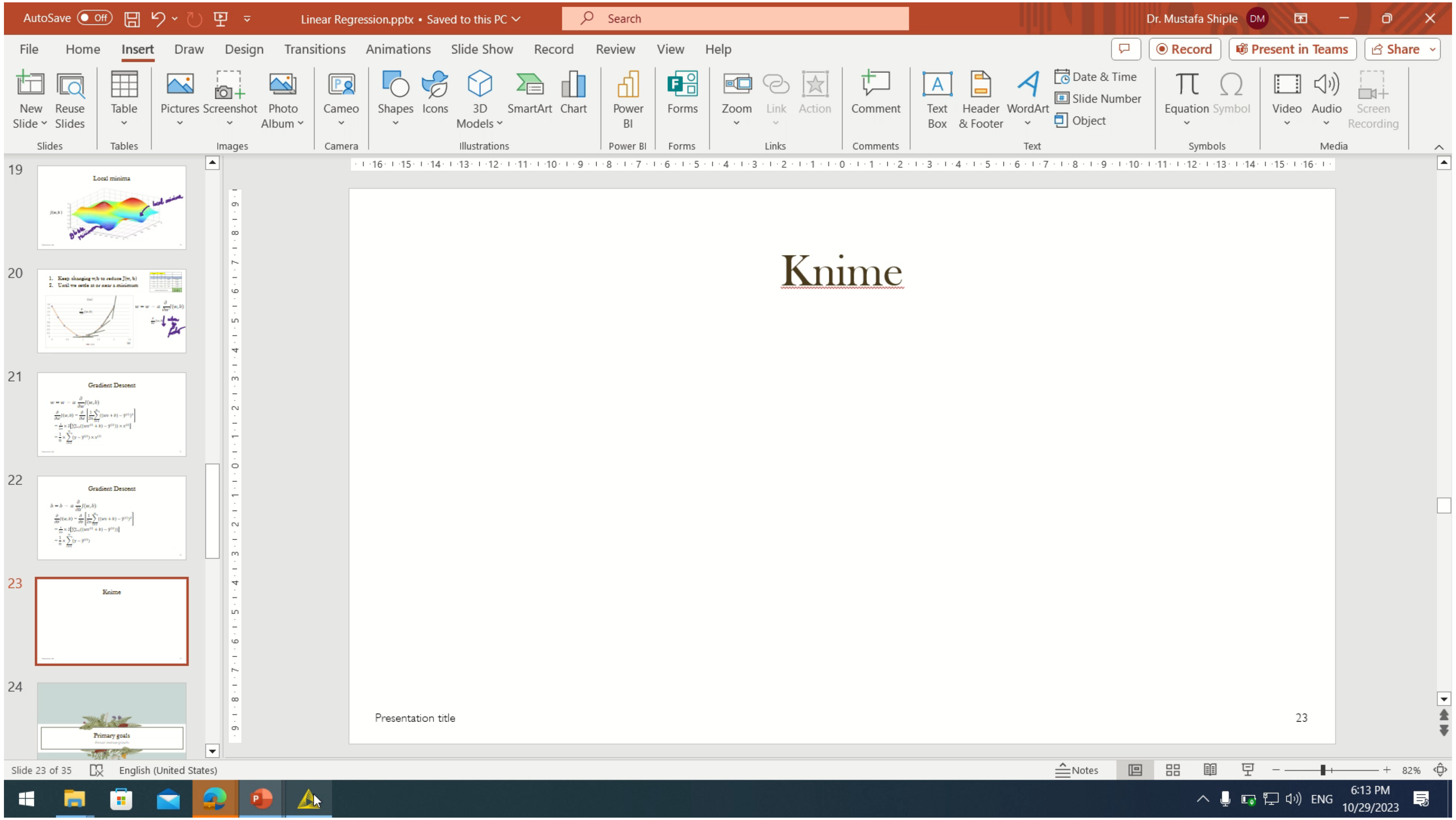
$$= \frac{1}{2n} \times 2\left[\sum_{i=1}^{n}((wx^{(i)} + b) - \hat{y}^{(i)})\right]$$

$$= \frac{1}{n} \times \sum_{i=1}^{n}(y - \hat{y}^{(i)})$$

# Batch gradient decent

- Batch gradient uses all training set data in learning phase.

# Knime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Linear Regression.pptx • Saved to this PC
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Record
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New Slide
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Shapes Icons 3D Models SmartArt Chart
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Symbols
Video Audio Screen Recording
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19
Local minima
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21
Gradient Descent
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23
Knime
24
Primary goals
Knime
Presentation title
23
Slide 23 of 35
English (United States)
Notes
82%
ENG
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10/29/2023

# The END