THE DIFFERENTIAL AMPLIFIER

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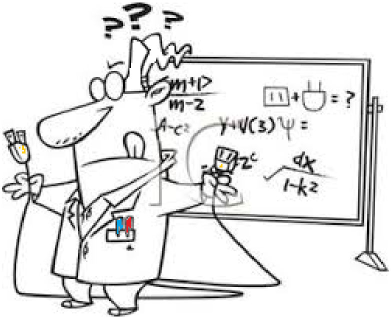
Advanced Electronic Circuits (EEI 184), 2018

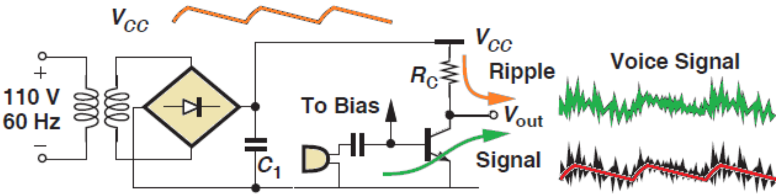
Outline

1. **Intro. to DA**
2. **Large Signal Analysis**
3. **Small Signal Analysis**
   - DA: Find Linear Region boundary

Intro. to DA



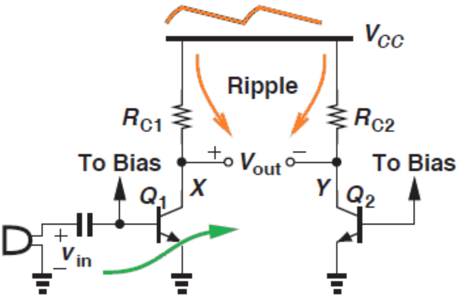
Problem



$$V_{out} = (V_{CC} + Noise) - i_C R_C \Rightarrow= (V_{CC} - i_C R_C) + Noise$$

Hum example

First Glance Solution



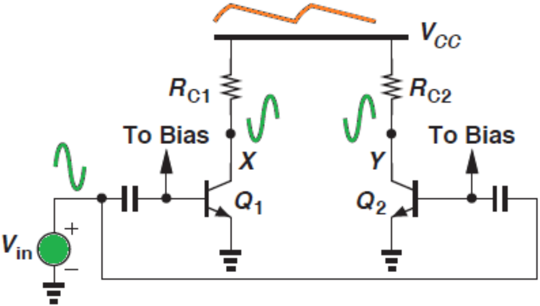
$$V_{out}^{+} = (V_{CC} - i_C R_C) + Noise \quad V_{out}^{-} = (V_{CC} - I_C R_C) + Noise$$

$$V_{out}^{+} - V_{out}^{-} = (V_{CC} - i_C R_C) + Noise - [(V_{CC} - I_C R_C) + Noise]$$

$$V_{out} = -i_C R_C + I_C R_C = -(I_C + i_c) R_C = -i_C R_C$$

Duplicate stage consisting of Q2 and RC2 remains "idle", thereby "wasting" current.

Improve Solution (Common Mode)

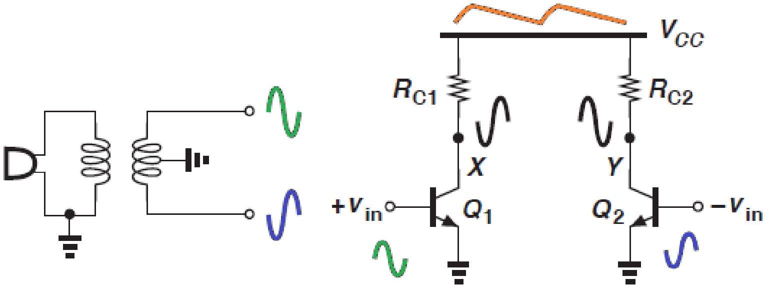


$$V_{out}^{+} = (V_{CC} - i_C R_C) + Noise \quad V_{out}^{-} = (V_{CC} - i_C R_C) + Noise$$

$$V_{out}^{+} - V_{out}^{-} = (V_{CC} - i_C R_C) + Noise - [(V_{CC} - i_C R_C) + Noise]$$

$$V_{out} = 0$$

Cont. Improve Solution (Differential Mode)



$$V^+_{out} = (V_{CC} - i_C R_C) + Noise \quad V^-_{out} = (V_{CC} - (-i_C)R_C) + N$$

$$V^+_{out} - V^-_{out} = (V_{CC} - i_C R_C) + Noise - [(V_{CC} - (-i_C)R_C) + Noise]$$

$$V_{out} = -i_c R_C - i_c R_C = -2 i_c R_C$$

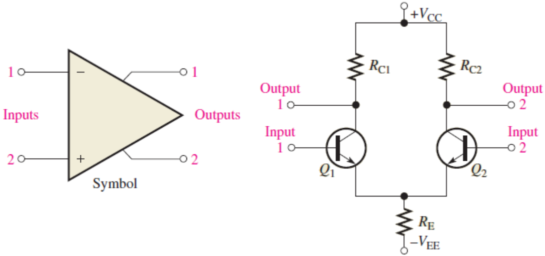
Definition

Differential Amplifier

is an amplifier that produces outputs that are a function of the difference between two input voltages.

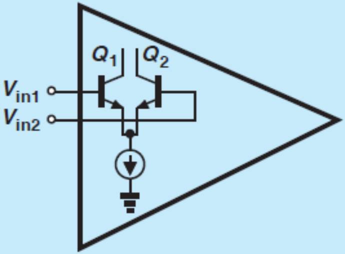
DA Modes:

- Two inputs are different (Differential).
- Two inputs are the same (Common mode).

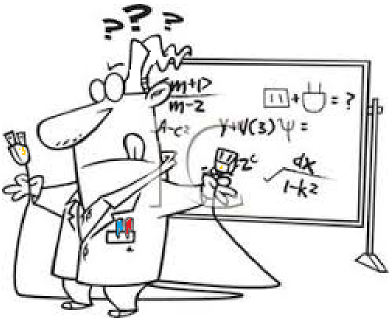


Did you know?

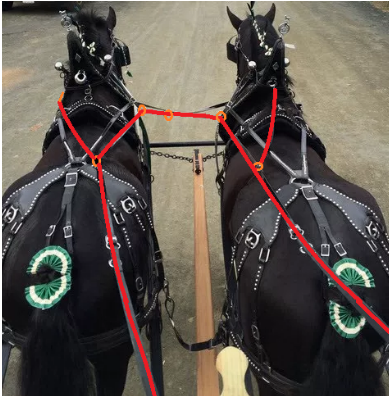
An important application of the differential pair is at the input of op amps, where inverting and noninverting input terminals are necessary. Without this second input, many op-amp-based functions would be difficult to realize. For example, the noninverting amplifier utilize both inputs.

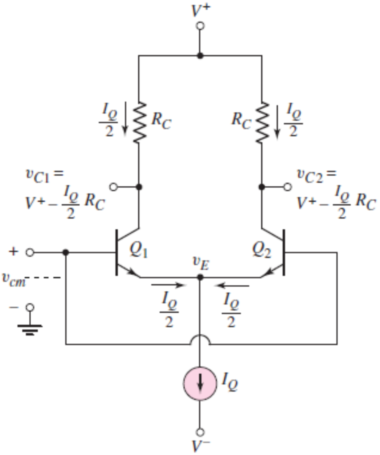


Large Signal Analysis



I_Q





"tail current source","emitter-coupled pair" or the "long-tailed pair."

large-signal operation

$$i_{C1} = I_S e^{v_{BE1}/V_T}$$

$$i_{C2} = I_S e^{v_{BE2}/V_T}$$

$$I_Q = i_{C1} + i_{C2}$$

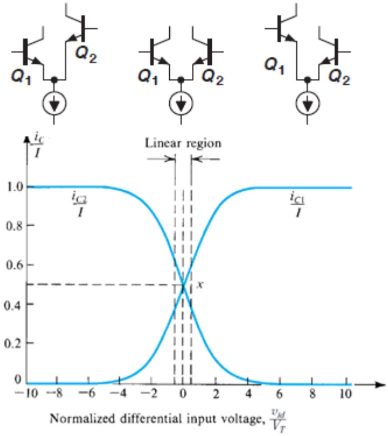
$$= I_S \left(e^{v_{BE1}/V_T} + e^{v_{BE2}/V_T} \right)$$

$$\frac{i_{C1}}{I_Q} = \frac{1}{1 + e^{(v_{BE2} - v_{BE1})/V_T}}$$

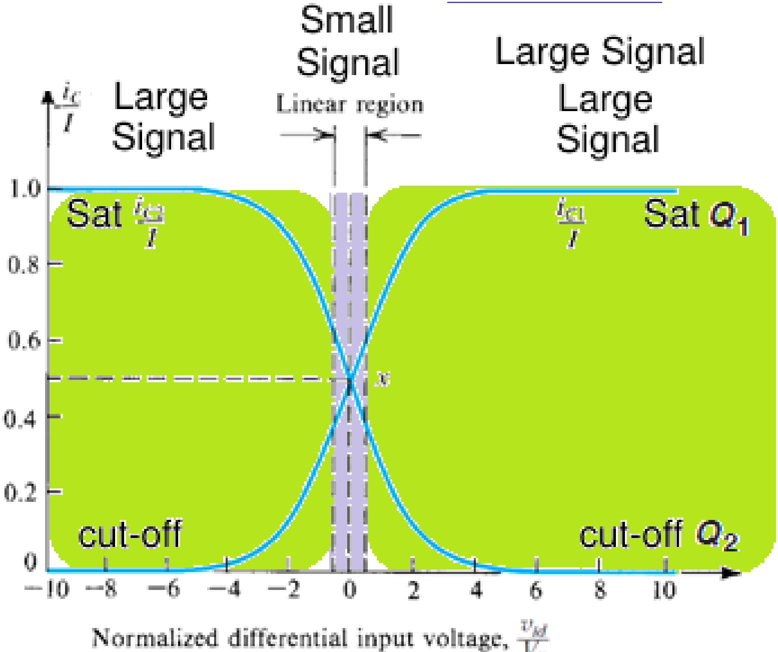
$$\frac{i_{C2}}{I_Q} = \frac{1}{1 + e^{-(v_{BE2} - v_{BE1})/V_T}}$$

$$\because v_{BE2} - v_{BE1} = v_d$$

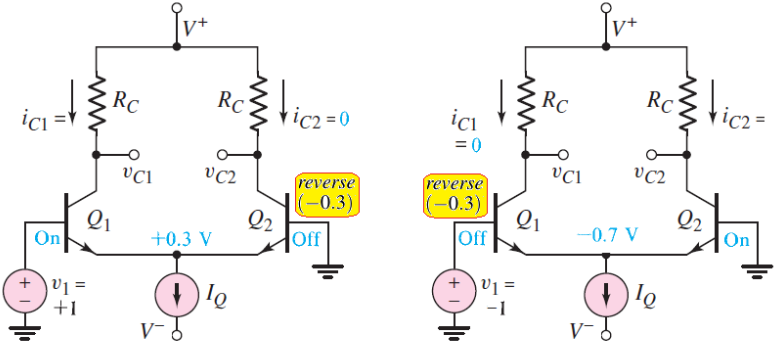
$$c\frac{i_{C2}}{I_Q} = \frac{1}{1 + e^{-(v_d)/V_T}}$$



Differential BJT



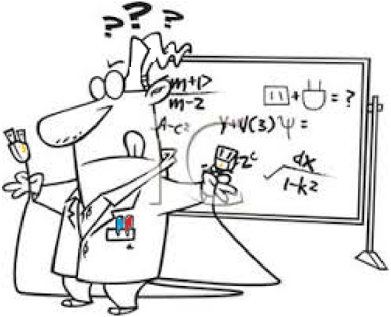
Large Signal Example



$$V_E = V_B - V_{BE}$$
$$= 1 - 0.7 = 0.3$$ to be active
$$\therefore V_{BE2} = -0.3$$ *Reverse*

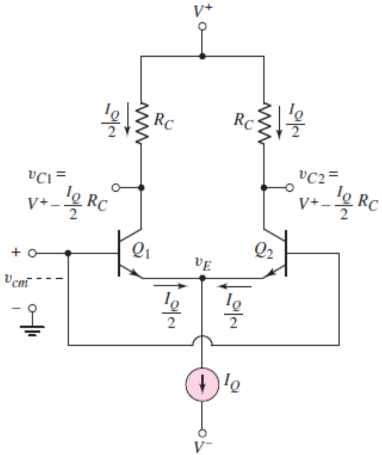
$$V_E = V_B - V_{BE}$$
$$= 0 - 0.7 = -0.3$$
$$\therefore V_{BE1} = -0.3$$

Small Signal Analysis



Active Differential BJT

Active means both transistors in active region, linear region, difference input voltages is small signal.
∵ transistors are matched,



$$v_{BE1} = v_{BE2}$$

$$i_{E1} = i_{E2} = \frac{I_Q}{2}$$

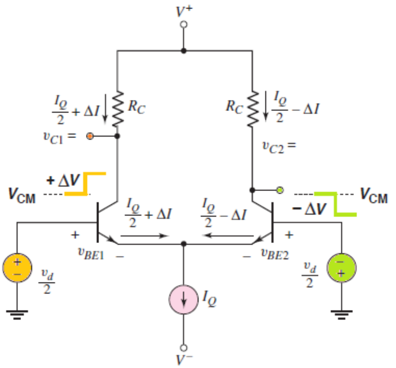
$$i_{C1} = i_{C2} \qquad \because i_C \approx i_E$$

$$\therefore v_{C1} = V^+ - \frac{I_Q}{2} R_C = v_{C2}$$

$$v_d = V_{C1} - V_{C2} = 0\#$$

Active Differential BJT

Two inputs are equal and inverted



$$v_{in1} = -v_{in2}$$
$$v_d = v_{in1} - v_{in2}$$
$$v_d = v_{in1} + v_{in1}$$
$$\Rightarrow v_{in1} = -v_{in2} = \frac{v_d}{2}$$

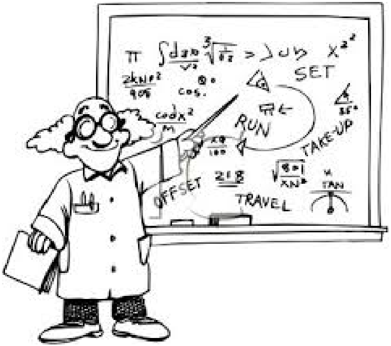
$$\therefore I_{E1} = \frac{I_Q}{2} + \Delta I \qquad I_{E2} = \frac{I_Q}{2} - \Delta I$$

$$v_{C1} = V^+ - (\frac{I_Q}{2} + \Delta I)R_C \qquad v_{C2} = V^+ - (\frac{I_Q}{2} - \Delta I)R_C$$

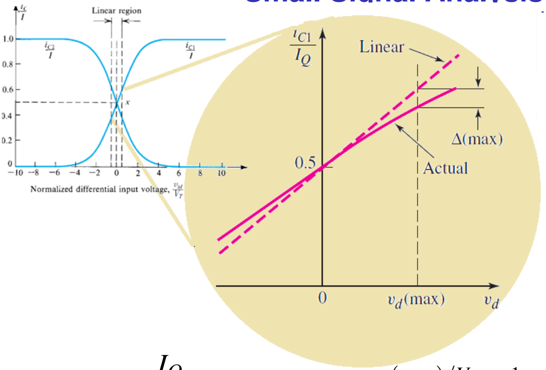
$$v_d = V_{C1} - V_{C2} = 2\Delta I R_C$$

ΔI : proportional to the difference input voltage

DA: Find Linear Region boundary



Small Signal Analysis

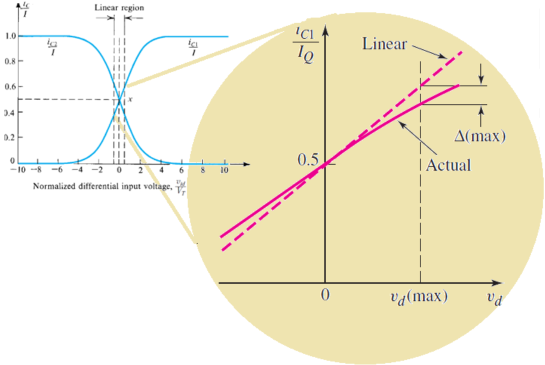


$$i_{C1} = \frac{I_Q}{1 + e^{(-v_d)/V_T}} = I_Q(1 + e^{(-v_d)/V_T})^{-1}$$

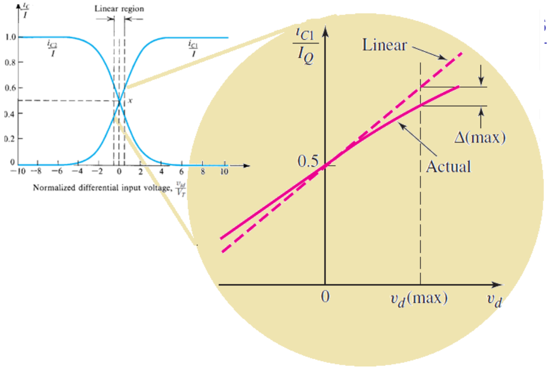
$$\frac{di_{C1}}{d(v_d)} = I_Q(-1)(1 + e^{(v_d)/V_T})^{-2}\left(\frac{-1}{V_T}e^{(-v_d)/V_T}\right)$$

$$= \frac{I_Q e^{(-v_d)/V_T}}{V_T(1 + e^{(-v_d)/V_T})^2}\Big|_{v_d=0} = \frac{I_Q}{4V_T}$$

Small Signal Analysis



Therefore the equation of a line passes through $I_c = 0.5I_Q$ and slop $\frac{I_Q}{4V_T}$ is $I_c = 0.5I_Q + \left(\frac{I_Q}{4V_T}\right) v_d$

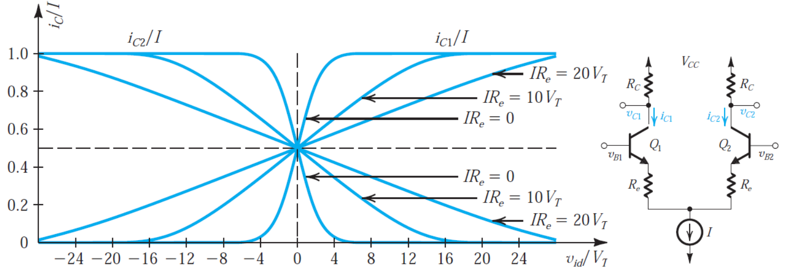


$$\frac{i_{C(linear)} - i_{C(actual))}}{i_{C(linear)}} = 0.01$$

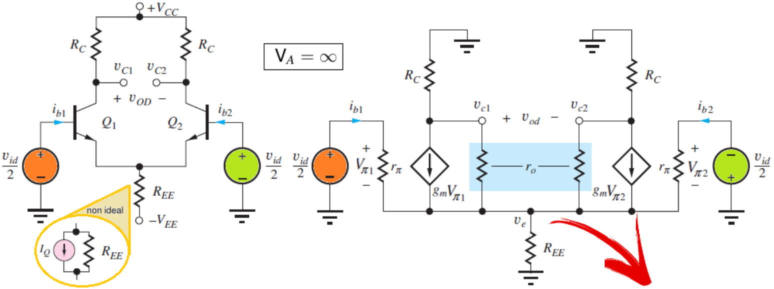
or

$$\frac{\left(0.5I_Q + \left(\frac{I_Q}{4V_T}\right)v_d\right) - \frac{I_Q}{1+e^{(-v_d)/V_T}}}{\left(0.5I_Q + \left(\frac{I_Q}{4V_T}\right)v_d\right)} = 0.01 \Rightarrow v_d \cong 18mv$$

Small Signal Analysis



Recall BJT AC analysis



Apply KCL @ node v_e

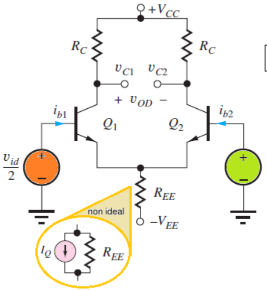
$$\because V_A = \infty \therefore r_o = \infty$$

$$g_m V_{\pi 1} + i_{b1} + g_m V_{\pi 2} + i_{b2} = \frac{v_e}{R_{EE}}$$

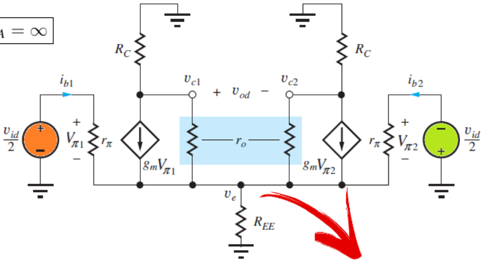
$$g_m V_{\pi 1} + \frac{V_{\pi 1}}{r_\pi} + g_m V_{\pi 2} + \frac{V_{\pi 2}}{r_\pi} = \frac{v_e}{R_{EE}}$$

$$(g_m + \frac{1}{r_\pi})(V_{\pi 1} + V_{\pi 2}) = \frac{v_e}{R_{EE}}$$

DA: Virtual Ground



$V_A = \infty$



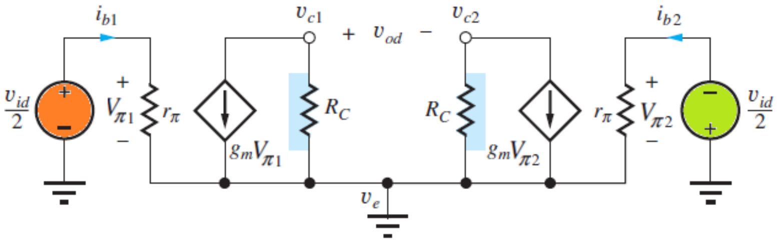
$$(g_m + \frac{1}{r_\pi})(V_{\pi1} + V_{\pi2}) = \frac{v_e}{R_{EE}}$$

$$(g_m + \frac{1}{r_\pi})(\frac{v_{id}}{2} - v_e + (\frac{-v_{id}}{2} - v_e)) = \frac{v_e}{R_{EE}}$$

$$(g_m + \frac{1}{r_\pi})(-2v_e) = \frac{v_e}{R_{EE}}$$

$$v_e(2g_m + 2\frac{1}{r_\pi} + \frac{1}{R_{EE}}) = 0$$

DA: Gain



$$v_{c1} = -g_m v_{\pi 1} R_C \qquad v_{c2} = -g_m v_{\pi 2} R_C$$

$$v_{c1} = -g_m \frac{v_{id}}{2} R_C \qquad v_{c2} = -g_m \frac{-v_{id}}{2} R_C$$

$$v_{od} = v_{c1} - v_{c2} = -g_m R_C v_{id}$$

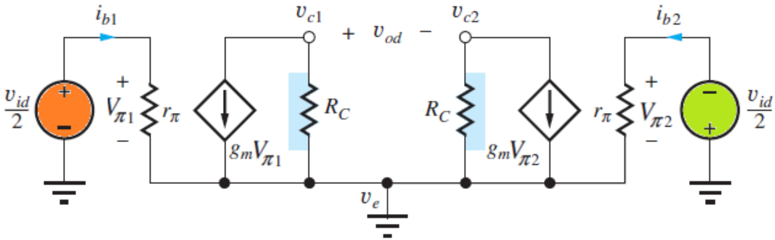
$$\boxed{Gain = \frac{v_{od}}{v_{id}} = -g_m R_C}$$

$$R_{id} = \frac{v_{id}}{i_{b1}} = 2r_\pi$$

$$R_{od} = 2R_C\big|_{differential}$$

$$R_{out} = R_C\big|_{\text{single ended}}$$

DA: Σ AC+ DC



$$i_c = -g_m v_{id} \qquad I_C = I_Q/2$$

$$v_C = V_{CC} - i_C R_C = V_{CC} - \frac{I_Q}{2} R_C + g_m v_{id} R_C$$

Example

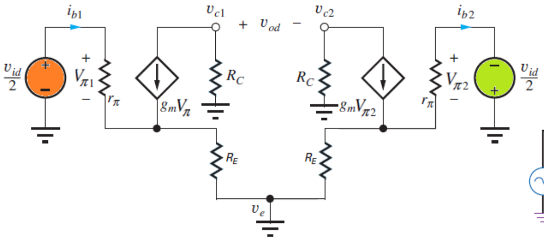
Design

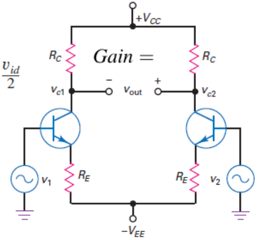
a bipolar differential pair for a gain of 10 and a power budget of 1 mW, 0.5 mW with a supply voltage of 2 V.

recall $P = IV \quad g_m = \frac{I_C}{V_T}$

$$I = \begin{cases} 0.5|_{1mW}, & I_C = 0.25mA, \quad g_m = 9.6m\mho, \quad R_C = 1041\Omega \\ 0.25|_{0.5mW}, & I_C = 0.125mA, \quad g_m = 4.8m\mho, \quad R_C = 2080\Omega \end{cases}$$

Low Input Resistance

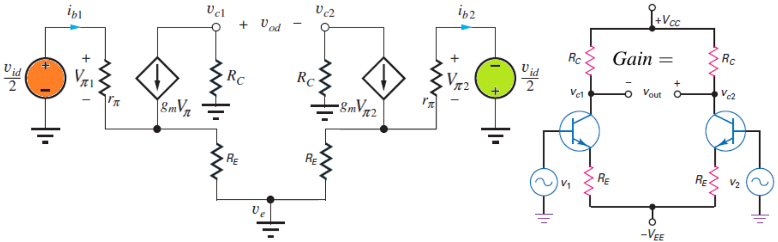




Recall $R_{id} = \dfrac{v_{id}}{i_{b1}} = 2r_\pi$

$$\frac{v_{id}}{2} = i_{b1} r_\pi + i_e R_E$$
$$\frac{v_{id}}{2} = i_{b1} r_\pi + (1+\beta) i_{b1} R_E$$
$$\frac{v_{id}}{i_{b1}} = 2(r_\pi + (1+\beta)R_E) = 2(r_\pi + (1+\beta)R_E) \uparrow$$

DA with R_E: Gain



Recall $Gain = -g_m R_C$ $\quad g_m r_\pi = \beta$

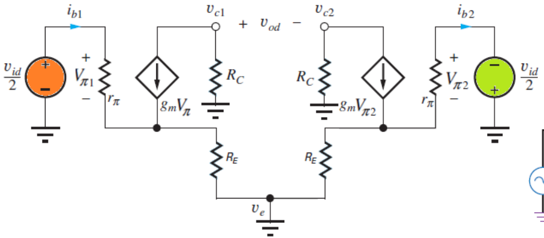
$$\frac{v_{id}}{2} = v_\pi + i_e R_E$$

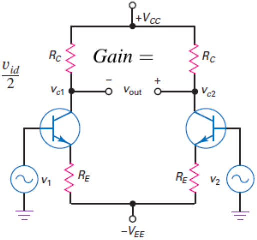
$$\frac{v_{id}}{2} = v_\pi + (\frac{v_\pi}{r_\pi} + g_m v_\pi) R_E$$

$$v_{id} = 2v_\pi \left(1 + (\frac{1}{r_\pi} + \frac{\beta}{r_\pi}) R_E\right) \Rightarrow v_\pi = \frac{v_{id}}{2(1 + \frac{1+\beta}{r_\pi} R_E)}$$

$$v_{out} = v_{c1} - v_{c2} = -g_m R_C (V_{\pi 1} - V_{\pi 2})$$

Cont. :DA with R_E: Gain





Recall $Gain = -g_m R_C \qquad g_m r_\pi = \beta$

$$v_{out} = v_{c1} - v_{c2} = -g_m R_C (V_{\pi 1} - V_{\pi 2})$$

$$v_{out} = \frac{-g_m R_C}{2(1 + \frac{1+\beta}{r_\pi} R_E)} (v_{id} - (-v_{id})) =$$

$$\boxed{Gain = \frac{-g_m R_C}{(1 + \frac{1+\beta}{r_\pi} R_E)}} \downarrow \Rightarrow \boxed{Gain = \frac{-\beta R_C}{r_\pi + (1+\beta) R_E} \approx \frac{-R_C}{R_E}}$$