Scaling and Transformation

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Shaping Circuits (EEC 242), 2015

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   - Frequency Scaling
   - Full Normalization
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Section 1

Scaling

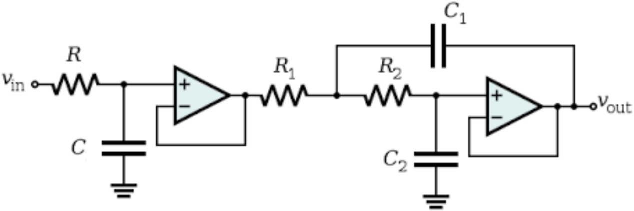
Subsection 1

Magnitude Scaling (Impedance Scaling)

Impedance Scaling

Magnitude scaling

is the process of increasing all impedances in a network by a factor, the frequency response remaining unchanged.



R	R_1	R_2	α_{max}	ω_c	C	C_1	C_2

$10^3.10^3.10^3.10^2.10.10^3=10^{15}$

The impedance of passive components

$$Z_R = R \quad Z_L = SL \quad Z_C = \frac{1}{SC}$$

Let the frequency remain constant

$$Z'_R = K_m R \quad Z'_L = K_m SL \quad Z'_C = K_m \frac{1}{SC}$$

$Therefore:$

$$R' = K_m R \quad L' = k_m L \quad C' = \frac{C}{K_m}$$

Test the frequency:

$$\omega'_o = \frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{k_m L \frac{C}{K_m}}}$$

$$\omega'_o = \frac{1}{\sqrt{LC}} = \omega_o$$

Subsection 2

Frequency Scaling

Frequency Scaling

is the process of shifting the frequency response of a network up or down the frequency axis while leaving the impedance the same.

The impedance of passive components

$$Z_R = R \text{ (not affected by frequency)} \quad Z_L = j\omega L \quad Z_C = \frac{1}{j\omega C}$$

$$Z'_L = j(\omega K_f)L \quad Z'_C = \frac{1}{j(\omega K_f)C}$$

Therefore :

$$R' = R \quad L' = \frac{L}{K_f} \quad C' = \frac{C}{K_f}$$

Test the frequency:

$$\omega'_o = \frac{1}{\sqrt{L'C'}} = \frac{1}{\sqrt{\frac{L}{K_f}\frac{C}{K_f}}}$$

$$\omega'_o = \frac{K_f}{\sqrt{LC}} = K_f\omega_o$$

Subsection 3

Full Normalization

Magnitude and Frequency Scaling

The general normalize rule:

$$R = \frac{R}{k_m} \quad L = \frac{K_f}{K_m} L' \quad C = K_m K_f C'$$

The general De-normalize rule:

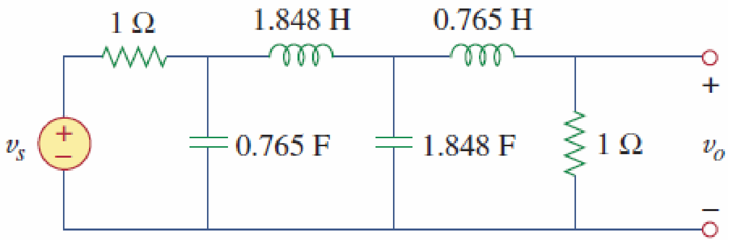
$$R' = k_m R \quad L' = \frac{K_m}{K_f} L \quad C' = \frac{1}{K_m K_f} C$$

Subsection 4

Normalization Examples

Magnitude and Frequency Scaling

Q1: A fourth order lowpass filter is shown. The filter is designed such that the cutoff frequency $\omega_o = 1 rad/s$. Scale the circuit for a cutoff frequency of $50kHz$ using $10k\Omega$ resistors.



Magnitude and Frequency Scaling

Q1: A fourth order lowpass filter is shown. The filter is designed such that the cutoff frequency $\omega_o = 1 rad/s$. Scale the circuit for a cutoff frequency of $50kHz$ using $10k\Omega$ resistors.

$$\omega_c' = 2\pi(50k) = 100k\pi$$

$$k_f = \frac{\omega_c'}{\omega_o} = \frac{100k\pi}{1} = \pi \times 10^5 \quad k_m = \frac{10k}{1} = 10k$$

$$Therefore: R' = k_m R = 10k\Omega$$

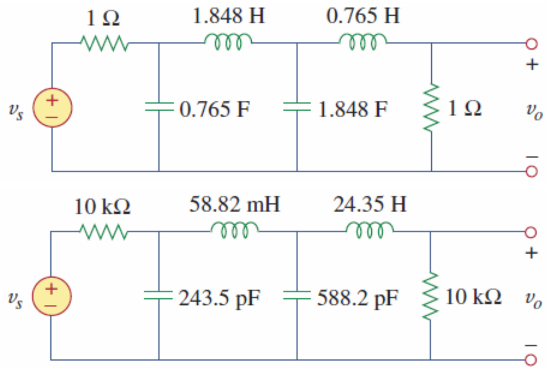
$$L_1' = \frac{K_m}{k_f} L = \frac{10^4}{\pi \times 10^5}(1.848) = 58.82mH$$

$$L_2' = \frac{10^4}{\pi \times 10^5}(0.765) = 24.35mH$$

$$C_1' = \frac{1}{K_f K_m} C = \frac{1}{\pi \times 10^9} 0.765 = 243.5pF$$

$$C_2' = \frac{1}{K_f K_m} C = \frac{1}{\pi \times 10^9} 1.848 = 588.2pF$$

Example 1


1 Ω
1.848 H
0.765 H
+
v_s
0.765 F
1.848 F
1 Ω
v_o
–
10 kΩ
58.82 mH
24.35 H
+
v_s
243.5 pF
588.2 pF
10 kΩ
v_o
–

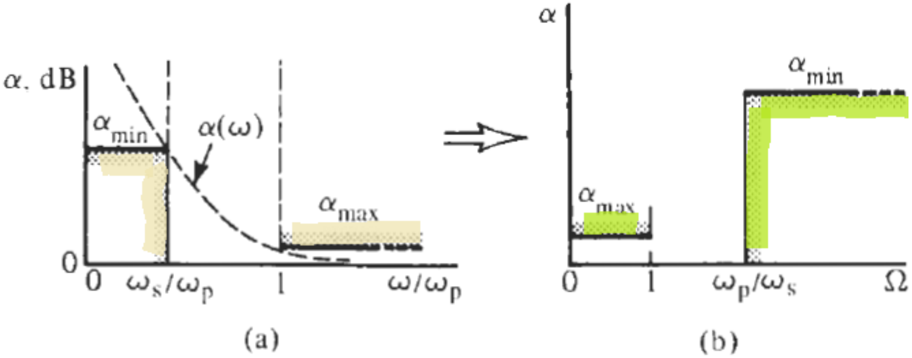
Section 2

Frequency Transformation

Subsection 1

Lowpass-to-Highpass Transformation

LP-HP procedure:

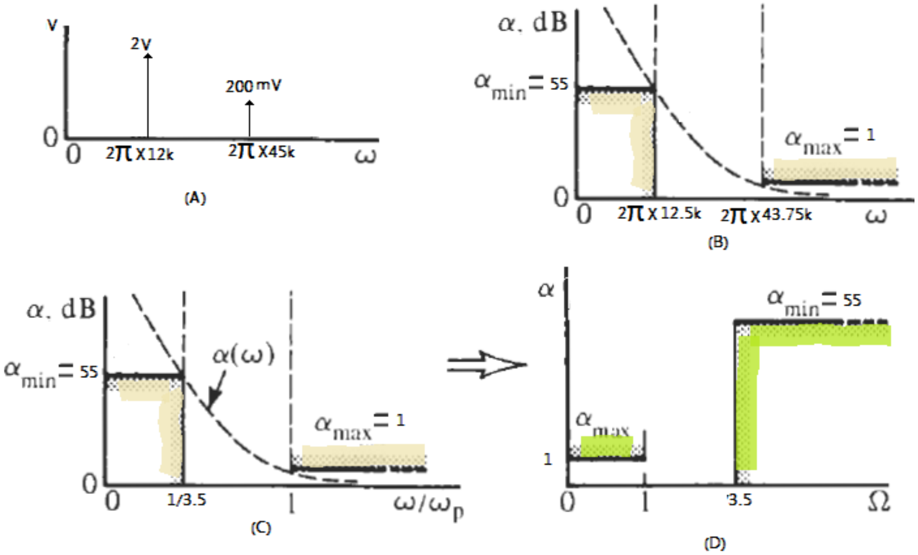


1. Normalize the frequency axis of HPF by passband corner frequency ω_p.
2. Use $\Omega = \frac{1}{\omega}$ and consider the attenuation values.
3. Find the degree n of LPF.
4. Compute the LPF transfer function $T_L(S)$.
5. Compute the HPF transfer function $T_H(S)$ by replacing S by $\frac{1}{S}$.
6. Realize $T_H(S)$.

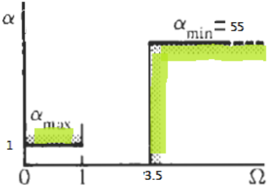
Subsection 2

Examples

Example 1:


V
2V
200 mV
0
0
2π X12k
2π X45k
ω
(A)
α, dB
α_min = 55
α_max = 1
0
0
2π X 12.5k
2π X43.75k
ω
(B)
α, dB
α_min = 55
α(ω)
α_max = 1
0
0
1/3.5
1
ω/ω_p
(C)
α
α_min = 55
α_max
1
0
1
3.5
Ω
(D)

Example 1:



3

$$n = \frac{\log\left(\frac{(10^{\frac{\alpha_{min}}{10}}-1)}{(10^{\frac{\alpha_{max}}{10}}-1)}\right)}{2\log\frac{\omega_S}{\omega_P}} = 5.5 \approx 6 \quad (1)$$

$$\varepsilon = \sqrt{(10^{\frac{\alpha_{max}}{10}} - 1)} = 0.51 \quad (2)$$

$$\omega_B = \varepsilon^{\frac{-1}{n}}\omega_p = 1.12 rad/sec \quad (3)$$

$$R_1 = R_2 = \frac{1}{\omega_B C} = 0.89\Omega \qquad \text{assume } c = 1F \quad (4)$$

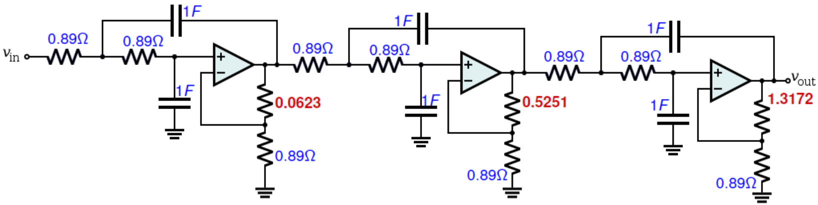
Example 1 (Cont.):

4

$$T_L(S) = \frac{1}{(S^2 + 1.93S + 1)(S^2 + 1.41S + 1)(S^2 + 0.52S + 1)}$$

.

$\frac{1}{Q}$	Q_n	K	$R_F = (k-1)R$
1.93	0.518135	1.07	0.0623
1.41	0.70922	1.59	0.5251
0.52	1.923077	2.48	1.3172



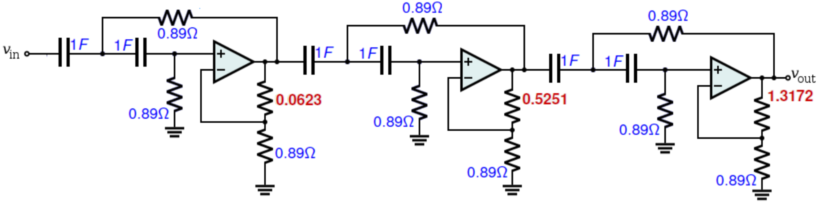
Example 1 (Cont.):

5

$$T_H(S) = \frac{1}{(S^{-2} + 1.93S^{-1} + 1)(S^{-2} + 1.41S^{-1} + 1)(S^{-2} + 0.52S^{-1} + 1)} \times (S^2 S^2 S^2)$$

$$T_H(S) = \frac{S^6}{(1 + 1.93S + S^2)(S^{-2} + 1.41S + S^2)(S^{-2} + 0.52S + S^2)}$$

.

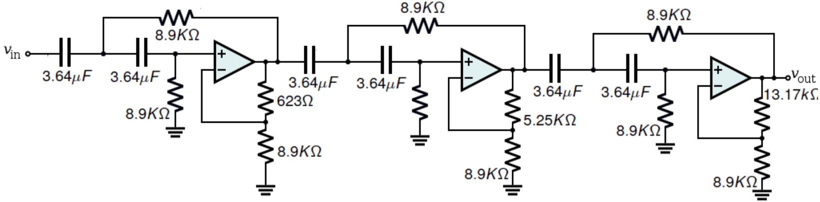


6 Realize $T_H(S)$.

$R = k_m R \Rightarrow R = 8.9K\Omega \quad R_{f1} = 623\Omega \quad R_{f2} = 5.25K\Omega \quad R_{f1} = 13.17k\Omega \quad \because K_M = 10K$

$C = \frac{1}{K_m K_F} C = \frac{1}{10k \cdot 2\pi 43.75k} = 3.64\mu F$

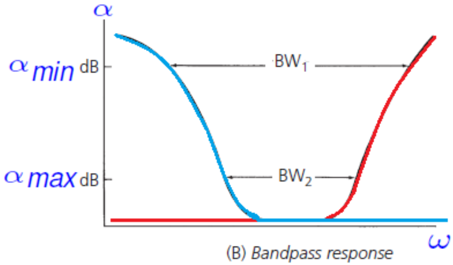
Example 1 (Cont.):



Subsection 3

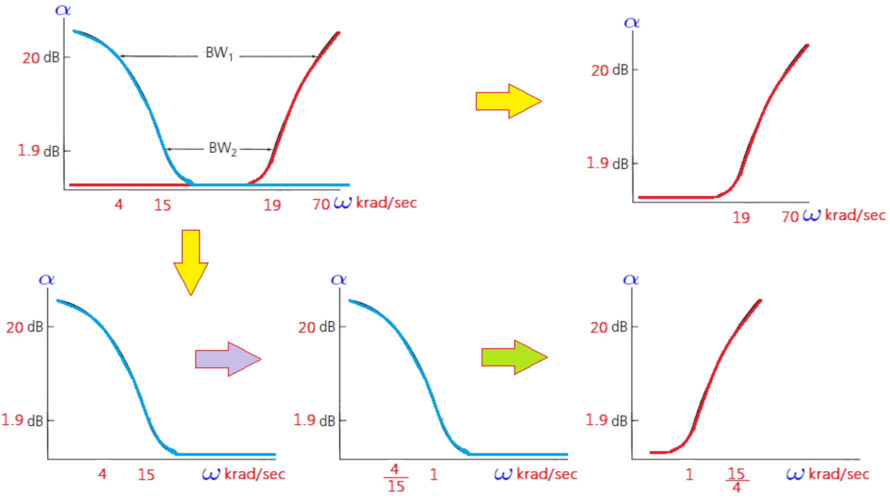
Lowpass-to-Bandpass Transformation

LP-BP procedure:



1. Discriminate between HPF and LPF.
2. Normalize the frequency axis of HPF by passband corner frequency ω_p.
3. Use $\Omega = \frac{1}{\omega}$ and consider the attenuation values.
4. Find the degree n of LPF.
5. Compute the LPF transfer function $T_L(S)$.
6. Compute the HPF transfer function $T_H(S)$ by replacing S by $\frac{1}{S}$.
7. Realize $T_H(S)$.

Example 1:


α
20 dB
1.9 dB
BW1
BW2
4
15
19
70 ω krad/sec
α
20 dB
1.9 dB
19
70 ω krad/sec
α
20 dB
1.9 dB
4
15
ω krad/sec
α
20 dB
1.9 dB
4/15
1
ω krad/sec
α
20 dB
1.9 dB
1
15/4
ω krad/sec

Section 3

Ladder filter

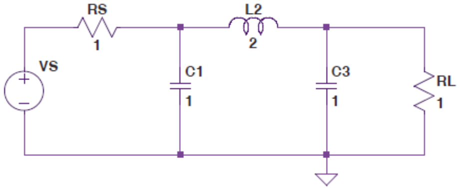
Normalized Filters

Order	R_S	C_1 a_1	L_2 a_2	C_3 a_3	L_4 a_4	C_5 a_5	L_6 a_6	C_7 a_7
1	1.0	2.0000						
2	1.0	1.4142	1.4142					
3	1.0	1.0000	2.0000	1.0000				
4	1.0	0.7654	1.8478	1.8478	0.7654			
5	1.0	0.6180	1.6180	2.0000	1.6180	0.6180		
6	1.0	0.5176	1.4142	1.9319	1.9319	1.4142	0.5176	
7	1.0	0.4450	1.2470	1.8019	2.0000	1.8019	1.2470	0.4450

Normalized Filters

Table: Filter Table

Order	RS	C1	L2	C3	L4
3	1	1	2	1	

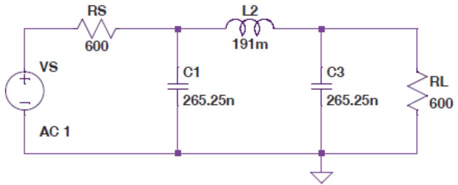


Examples

Design a Butterworth LPF of order $n = 3$ with $-3dB$ frequency $fr = 1kHz$ and $RS = RL = 600\Omega$.

Solution: We find that $\omega_r = 2\pi 1000 = 6283.2 rad/s$.

Order	k_m	k_f	Rule	Realization
RS	600	6283.2	$k_m R_n$	600Ω
C1	600	6283.2	$\frac{1}{k_m k_f} L_n$	$265.25nF$
L2	600	6283.2	$\frac{k_m}{k_f} L_n$	$191mH$
C3	600	6283.2	$\frac{1}{k_m k_f} L_n$	$265.25nF$
RL	600	6283.2	$k_m R_n$	600Ω



Subsection 1

Filter Transformation

LPF to HPF

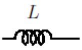
The substitution $s = \frac{1}{s}$ affects the C and L elements of the LPF as follows:

$$Z_C = \frac{1}{sC} \Rightarrow Z_C = \frac{1}{\frac{1}{s}C} = s\frac{1}{C} \quad (C \Rightarrow L = \frac{1}{C})$$

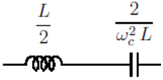
$$Z_L = sL \Rightarrow Z_C = \frac{1}{s}L = \frac{L}{s} \quad (L \Rightarrow C = \frac{1}{L})$$

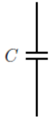
LPF to BPF

The substitution $s = \frac{s^2+\omega_c^2}{2s}$ affects the C and L elements of the LPF as follows:

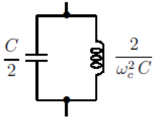


$Ls \implies L\frac{s^2+\omega_c^2}{2s} = \frac{Ls}{2} + \frac{\omega_c^2 L}{2s}$



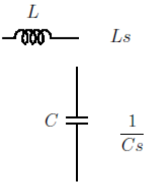


$\frac{1}{Cs} \implies \frac{1}{C\frac{s^2+\omega_c^2}{2s}} = \frac{1}{\frac{Cs}{2} + \frac{\omega_c^2 C}{2s}}$



Example

Example: Butterworth BPF of order n = 6 with bandwidth 100 kHz, center frequency $f_c = 1MHz$, and $R_S = R_L = 50$. Start from a normalized Butterworth LPF of order n = 3 with $C_1 = C_3 = 1F$and $L_2 = 2H$. Then use frequency and impedance scaling factors



$$\Longrightarrow \quad L\frac{s^2+\omega_c^2}{2s} = \frac{Ls}{2} + \frac{\omega_c^2 L}{2s}$$

$$\Longrightarrow \quad \frac{1}{C\frac{s^2+\omega_c^2}{2s}} = \frac{1}{\frac{Cs}{2} + \frac{\omega_c^2 C}{2s}}$$