Second Order Biquadratic topologies

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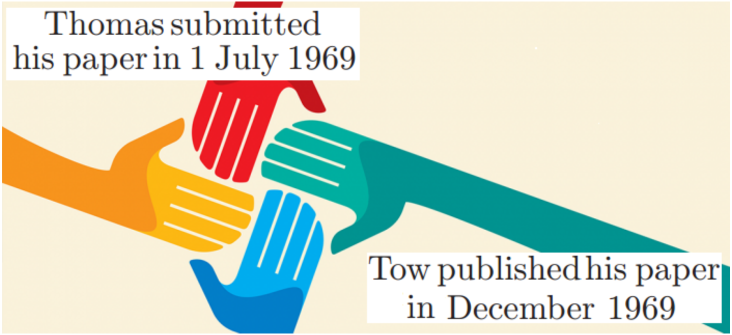
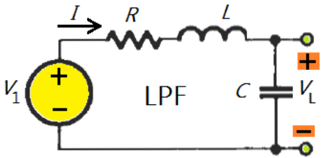
Shaping Circuits (EEC 242), 2015

Outline

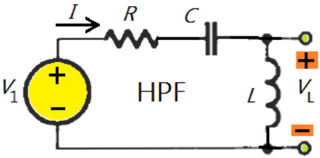
Introduction



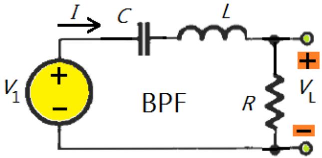
$H(S)$ $\qquad$ $\phi(j\omega)$



$$= \frac{\omega_0^2}{S^2 + S\frac{\omega_0}{Q} + \omega_0^2} \qquad = -\tan^{-1}\frac{\omega\frac{\omega_0}{Q}}{\omega_0^2 - \omega^2}$$

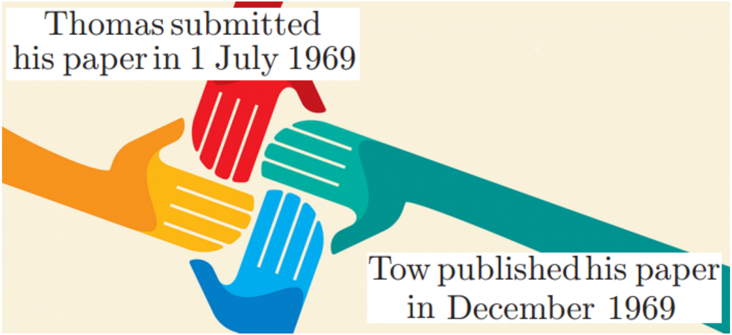


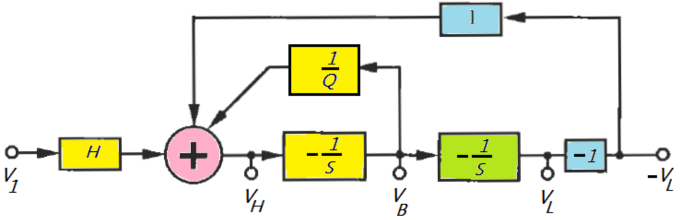
$$= \frac{s^2}{S^2 + S\frac{\omega_0}{Q} + \omega_0^2} \qquad = 180 - \tan^{-1}\frac{\omega\frac{\omega_0}{Q}}{\omega_0^2 - \omega^2}$$

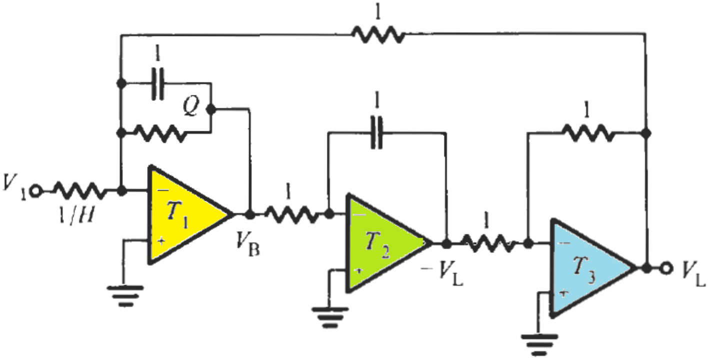


$$= \frac{S\frac{\omega_0}{Q}}{S^2 + S\frac{\omega_0}{Q} + \omega_0^2} \qquad = 90 - \tan^{-1}\frac{\omega\frac{\omega_0}{Q}}{\omega_0^2 - \omega^2}$$

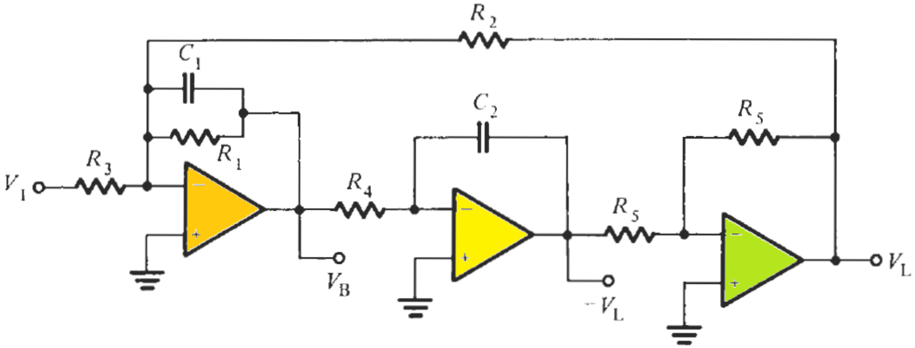
Tow Thomas


Thomas submitted
his paper in 1 July 1969
Tow published his paper
in December 1969


1
1
Q
H
+
- 1
s
- 1
s
-1
V_1
V_H
V_B
V_L
$-V_L$

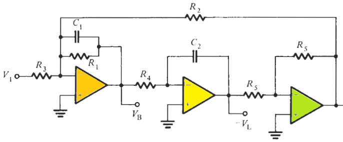

1
1
Q
1
1
V_1
1/H
T_1
V_B
1
T_2
$-V_L$
1
T_3
V_L

wiring diagram



Wiring Diagram:

$$V_B = -\frac{Z_F}{Z}V_1 - \frac{Z_F}{Z}V_L$$



$$Z_F = (Z_{C_1} \| Z_{R_1}) = \frac{R_1}{1 + SC_1R_1}$$

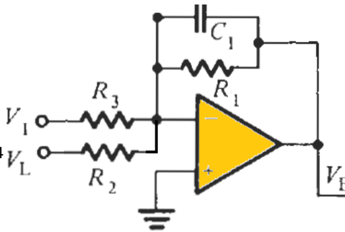
$$(1 + SC_1R_1)V_B = -\frac{R_1}{R_3}V_1 - \frac{R_1}{R_2}V_L$$

$$\because V_L = \frac{1}{SC_2R_4}V_B$$

$$(1 + SC_1R_1 + \frac{R_1}{SC_2R_2R_4})V_B = -\frac{R_1}{R_3}V_1$$

$$\frac{V_B}{V_1} = \frac{-\frac{R_1}{R_3}SC_2R_2R_4}{SC_2R_2R_4 + S^2C_1C_2R_1R_2R_4 + R_1}$$

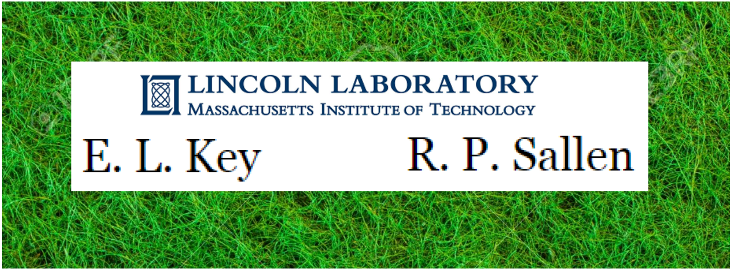
$C_1C_2R_1R_2R_4$



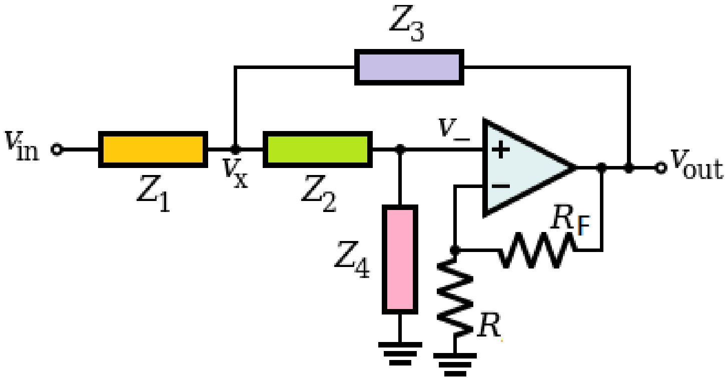
$$\frac{V_B}{V_1} = \frac{\left(-\frac{R_1}{R_3}\right)S\frac{1}{C_1R_1}}{S^2 + S\frac{1}{C_1R_1} + \frac{1}{C_1C_2R_2R_4}}$$

$$\therefore \omega_o = \frac{1}{\sqrt{C_1C_2R_2R_4}} \text{ and } Q = \frac{R_1}{\sqrt{R_2R_4}}\sqrt{\frac{C_1}{C_2}}$$

Sallen Key Topology

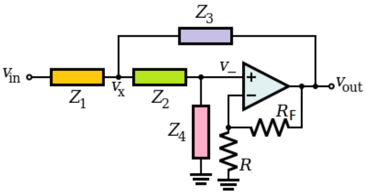


wiring diagram



KCL:

$$V^-(1+\frac{R_F}{R}) = V^-K = V_{out}$$



V_x :

$$\frac{V_{in}-V_x}{Z_1} = \frac{V_x-V^-}{Z_2} + \frac{V_x-V_{out}}{Z_3}$$

$$\frac{V_x-V^-}{Z_2} = \frac{V^-}{Z_4}$$

$$\therefore V_x = V^-\left(\frac{Z_2}{Z_4}+1\right)$$

$$\boxed{\frac{V_{out}}{V_{in}} = \frac{KZ_3Z_4}{Z_1Z_3+Z_1Z_2+Z_2Z_3+Z_1Z_4+Z_3Z_4-KZ_1Z_4}} \quad (1)$$

Cont.

V_x :

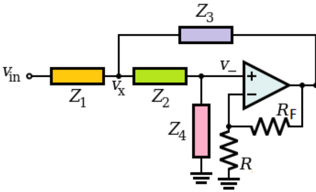
$$V_{in} - V_x = V_x \frac{Z_1}{Z_2} - V^{-} \frac{Z_1}{Z_2} + V_x \frac{Z_1}{Z_3} - V_{out} \frac{Z_1}{Z_3}$$

$$V_{in} = V_x \left(\frac{Z_1}{Z_2} + \frac{Z_1}{Z_3} + 1\right) - V^{-} \frac{Z_1}{Z_2} - V_{out} \frac{Z_1}{Z_3}$$

$$V_{in} = \frac{V_{out}}{K} \left(\frac{Z_2}{Z_4} + 1\right) \left(\frac{Z_1}{Z_2} + \frac{Z_1}{Z_3} + 1\right) - \frac{V_{out}}{K} \frac{Z_1}{Z_2} - V_{out} \frac{Z_1}{Z_3}$$

$$\frac{V_{in}}{V_{out}} = \frac{1}{K} \left(\frac{Z_1}{Z_4} + \frac{Z_1 Z_2}{Z_4 Z_3} + \frac{Z_2}{Z_4} + \cancel{\frac{Z_1}{Z_2}} + \frac{Z_1}{Z_3} + 1 - \cancel{\frac{Z_1}{Z_2}}\right) - \frac{Z_1}{Z_3}$$

$$= \frac{Z_1 Z_3 + Z_1 Z_2 + Z_2 Z_3 + Z_1 Z_4 + Z_3 Z_4 - K Z_1 Z_4}{K Z_3 Z_4}$$



Low Pass Filter

$$\frac{V_{out}}{V_{in}} = \frac{KZ_3Z_4}{Z_1Z_3 + Z_1Z_2 + Z_2Z_3 + Z_1Z_4 + Z_3Z_4 - KZ_1Z_4}$$

$$\frac{V_{out}}{V_{in}} = \frac{K\frac{1}{SC_3}\frac{1}{SC_4}}{R_1\frac{1}{SC_3} + R_1R_2 + R_2\frac{1}{SC_3} + R_1\frac{1}{SC_4} + \frac{1}{SC_3}\frac{1}{SC_4} - KR_1\frac{1}{SC_4}} \qquad \times S^2C_4C_3$$

$$= \frac{K}{R_1SC_4 + s^2C_3C_4R_1R_2 + R_2SC_4 + R_1SC_3 + 1 - KR_1SC_3} \qquad \div C_3C_4R_1R_2$$

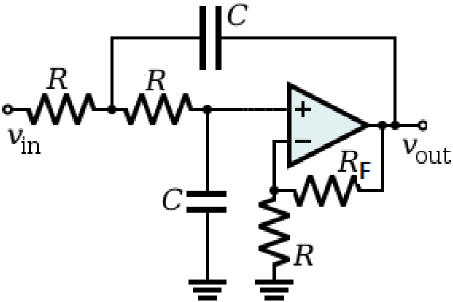
$$= \frac{\frac{K}{C_3C_4R_1R_2}}{s^2 + S\frac{R_1C_4 + R_2C_4 + R_1C_3 - KR_1C_3}{C_3C_4R_1R_2} + \frac{1}{C_3C_4R_1R_2}} \qquad C_3 = C_4 = C$$

Low Pass Filter

$$T(S) = \frac{\frac{K}{C^2 R_1 R_2}}{s^2 + S\frac{R_2 + R_1(2-K)}{CR_1 R_2} + \frac{1}{C^2 R_1 R_2}}$$

$$\omega_0 = \frac{1}{C\sqrt{R_1 R_2}}, Q = \frac{\sqrt{R_1 R_2}}{R_2 + R_1(2-K)} \qquad R_1 = R_2 = R$$

$$\omega_0 = \frac{1}{RC}, \quad Q = \frac{1}{(3-K)}, \quad K = 1 + \frac{R_F}{R}$$



$$\omega_0 = \frac{1}{RC}, \quad Q = \frac{1}{(3-K)}, \quad K = 1 + \frac{R_F}{R}$$

Sensetivity

- Sensitivity:

$$Q = \frac{1}{(3-K)}$$

$$\frac{dQ(K)}{dK} = \frac{1}{(3-K)^2} = Q^2$$

$$\therefore dQ(K) = Q^2 dK$$

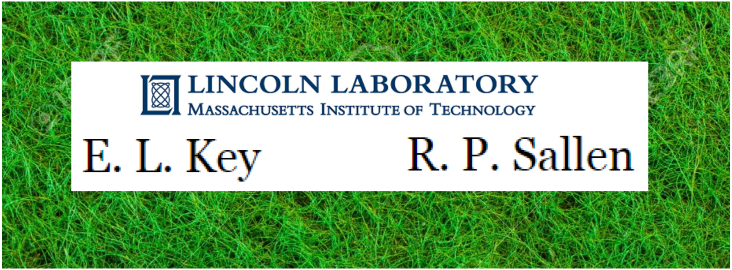
$$\frac{dQ(K)}{Q} = QdK \qquad \times \frac{K}{K}$$

$$\frac{dQ(K)}{Q} = KQ\frac{dK}{K}$$

- Gain adjustment:

$$K = 3 - \frac{1}{Q}$$

Sallan Key designs



Quality Factor (Q) or Damping Ratio(ζ)

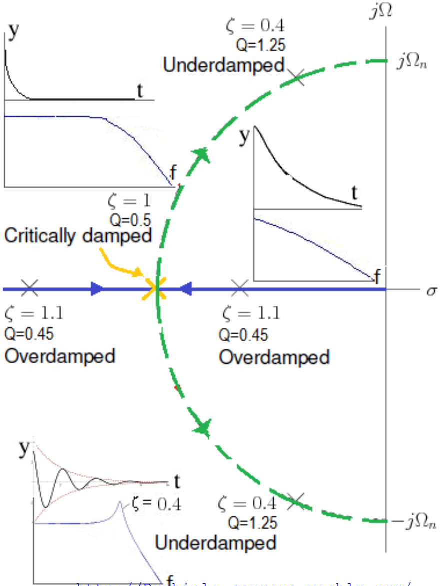
$$Q = \frac{1}{2\zeta}$$

$$H(S) = \frac{\omega_o^2}{S^2 + \frac{\omega_o}{Q}S + \omega_o^2} = \frac{\omega_o^2}{S^2 + 2\zeta\omega_o S + \omega_o^2}$$

$$S_{1,2} = -\zeta\omega_o \pm \omega_o\sqrt{\zeta^2 - 1} \qquad = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$S_{1,2} = \begin{cases} \zeta > 1 \text{ or } Q < 0.5, & \text{Distinct real roots} \quad \text{Overdamped } 1^{st} \, order \\ \zeta = 1 \text{ or } Q = 0.5, & \text{Identical real roots} \quad \text{Critically damped} \\ \zeta < 1 \text{ or } Q > 0.5, & \text{Distinct complex roots} \quad \text{Underdamped} \\ \zeta = 0 \text{ or } Q = \infty, & \text{Oscillator} \end{cases}$$

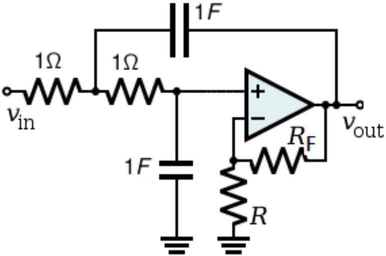
Root Locus of Q



Design 1/Simplification 1: Set Filter Components Equal

Features:

1. $1 < K < 3$
2. Avoid k=1 $\Rightarrow Q = 0.5$
3. Avoid k=3 $\Rightarrow Q = \infty$
4. ALL components are = 1,
   $R_1 = R_2 = 1\Omega \quad C_3 = C_4 = 1F$
5. Very sensitive.



$$T(S) = \frac{k\frac{1}{R_1R_2C_3C_4}}{S^2 + [\frac{1}{R_1C_3} + \frac{1}{R_2C_3} + \frac{1}{R_2C_4} - \frac{k}{R_2C_4}]S + \frac{1}{R_1R_2C_3C_4}}$$

assume $R_1 = R_2 = C_3 = C_4 = \omega_o = 1$

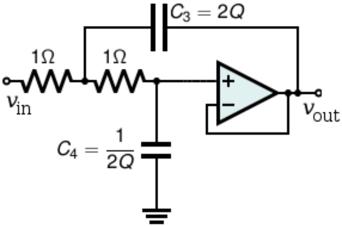
$$= \frac{k}{S^2 + [3 - K]S + 1}$$

$$K = 3 - \frac{1}{Q} = 1 + \frac{R_a}{R_b}$$

Design 2/Simplification 2:Set Filter Components as Ratios and Gain = 1

Features:

1. k=1
2. Q is independent on K.
3. $R_1 = R_2 = 1\Omega$ and $C_3 = \frac{1}{C_4}$



$$T(S) = \frac{k\frac{1}{R_1R_2C_3C_4}}{S^2 + [\frac{1}{R_1C_3} + \frac{1}{R_2C_3} + \frac{1}{R_2C_4} - \frac{k}{R_2C_4}]S + \frac{1}{R_1R_2C_3C_4}}$$

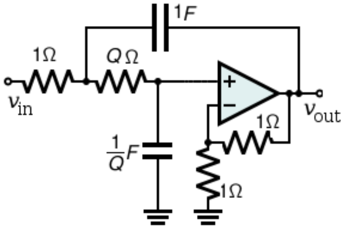
$$\text{assume } R_1 = R_2 = 1\Omega \quad \text{and } C_3 = \frac{1}{C_4}$$

$$= \frac{1}{S^2 + \frac{2}{C_3}S + 1} \quad \Rightarrow C_3 = 2Q \quad C_4 = \frac{1}{2Q}$$

Design 3/Simplification 3:Set Filter Components as Ratios and Gain = 2

Features:

1. k=2 $\Rightarrow R_a = R_b = 1\Omega$
2. Q is independent on K.
3. $R_1 = 1\Omega$, $C_3 = 1F$ and $R_2 = \frac{1}{C_4}$



$$T(S) = \frac{k\frac{1}{R_1R_2C_3C_4}}{S^2 + [\frac{1}{R_1C_3} + \frac{1}{R_2C_3} + \frac{1}{R_2C_4} - \frac{k}{R_2C_4}]S + \frac{1}{R_1R_2C_3C_4}}$$

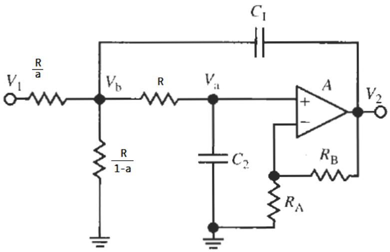
$$\text{assume } R_1 = 1\Omega \quad \text{and } R_2 = \frac{1}{C_4}$$

$$= \frac{2}{S^2 + \frac{1}{R_2}S + 1} \qquad \Rightarrow R_2 = Q = \frac{1}{C_4}$$

Resistive Gain Reduction

Features:

1. could be applied to all simplifications.

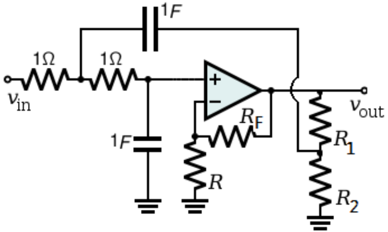


$$R_1 = (\frac{R}{a} \parallel \frac{R}{1-a})$$

Resistive Gain Enhancment

Features:

1. ALL components are = 1,
$R_1 = R_2 = 1\Omega \quad C_3 = C_4 = 1F$



$$T(S) = \frac{k\frac{1}{R_1R_2C_3C_4}}{S^2 + [\frac{1}{R_1C_3} + \frac{1}{R_2C_3} + \frac{1}{R_2C_4} - \frac{kk_d}{R_2C_4}]S + \frac{1}{R_1R_2C_3C_4}}$$

$$\text{assume } R_1 = R_2 = C_3 = C_4 = \omega_o = 1$$

$$= \frac{k}{S^2 + [3 - KK_d]S + 1} \Rightarrow K_d = \frac{R_2}{R_1 + R_2}$$

$$KK_d = 3 - \frac{1}{Q}$$