Second Order Analog Filters

Dr. M. Shiple

Shaping Circuits (EEC 242), 2015

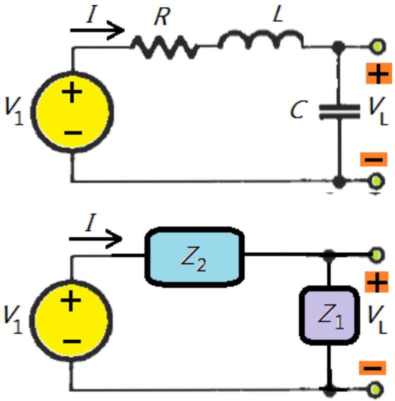
Outline

Example

- Find the transfer function.

$$T(S) = \frac{Z_1}{Z_1 + Z_2}$$

Hint: By changing Z_1, Numerator will take different forms and denominator will kept static.



cont.

$$H(S) = \frac{\frac{1}{Sc}}{R + SL + \frac{1}{Sc}}$$

$$SL + \frac{1}{Sc} = 0$$

$$H(S) = \frac{1}{S^2LC + SCR + 1} \quad \div LC$$

$$= \frac{\frac{1}{LC}}{S^2 + S\frac{R}{L} + \frac{1}{LC}}$$

$$= \frac{\omega_0^2}{S^2 + S\frac{\omega_0}{Q} + \omega_0^2}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad (1)$$

$$\text{Band width } = \beta = \frac{R}{L} \qquad (2)$$

$$\text{Quality factor} = Q = \frac{\omega_0}{\beta} \qquad (3)$$

Magnitude and phase

- Replace ALL S by $j\omega$

$$H(S) = \frac{\omega_0^2}{S^2 + S\frac{\omega_0}{Q} + \omega_0^2}$$

$$T(j\omega) = \frac{\omega_0^2}{(j\omega)^2 + j\omega\frac{\omega_0}{Q} + \omega_0^2}$$

$$|T(j\omega)| = \frac{\omega_0^2}{\sqrt{(\omega_0^2 - \omega^2)^2 + (\omega\frac{\omega_0}{Q})^2}}$$

$$\phi(j\omega) = -\tan^{-1}\frac{\omega\frac{\omega_0}{Q}}{\omega_0^2 - \omega^2}$$

Frequncy response (FS) graph

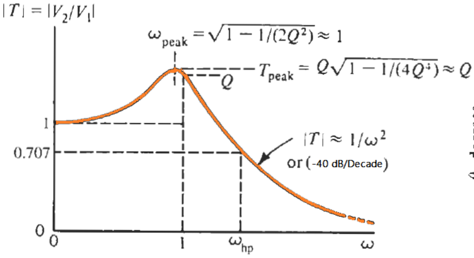
$$|T(j\omega)| = \frac{\omega_0^2}{\sqrt{(\omega_0^2-\omega^2)^2 + (\omega\frac{\omega_0}{Q})^2}}$$

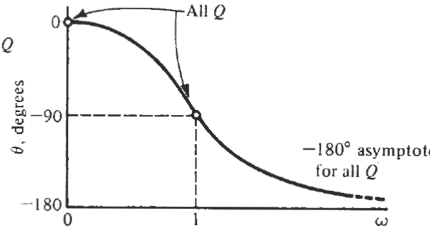
1. @ $\omega = zero \Rightarrow$
   $$|T(j\omega)| = \frac{\omega_0^2}{\sqrt{(\omega_0^2)^2}} = 1$$
2. @ $\omega = \omega_0 \Rightarrow$
   $$|T(j\omega)| = \frac{\omega_0^2}{\sqrt{(\frac{\omega_0^2}{Q})^2}} = Q$$

$$\phi(j\omega) = -\tan^{-1}\frac{\omega\frac{\omega_0}{Q}}{\omega_0^2-\omega^2}$$

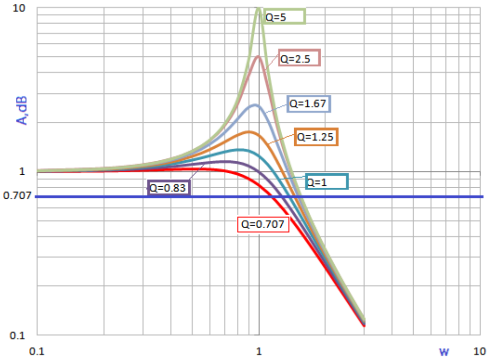
1. @ $\omega = zero \Rightarrow \phi = 0$
2. @ $\omega = \omega_0 \Rightarrow \phi = -90$
3. @ $\omega = \infty \Rightarrow \phi = -180$

Effect of Q


|T| = |V2/V1|
ωpeak = √1 − 1/(2Q²) ≈ 1
Tpeak = Q√1 − 1/(4Q⁴) ≈ Q
Q
1
0.707
0
0
1
ωhp
ω
|T| ≈ 1/ω²
or (-40 dB/Decade)

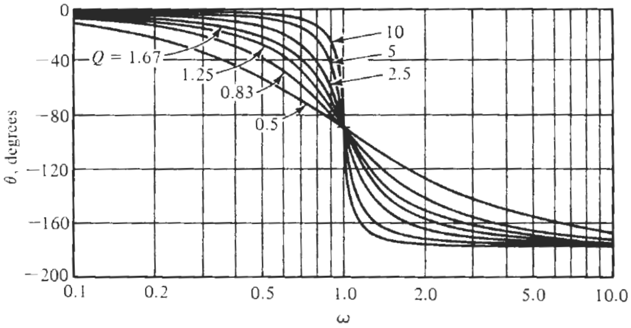

All Q
0
−90
−180
θ, degrees
−180° asymptote
for all Q
0
1
ω

Effect of Q (Magnitude)



Hint: at $Q = 0.707$ (Flat magnitude) $\omega_0 = \omega_c$

Effect of Q (phase)



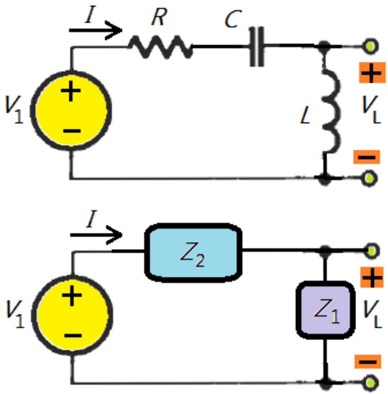
Hint: @ ALL Q $\phi = 0$ @ $\omega = 0$ and $\phi = -90$ @ $\omega = \omega_0$

Example

- Find the transfer function.

$$T(S) = \frac{Z_1}{Z_1 + Z_2}$$

Hint: By changing Z_1, Numerator will take different forms and denominator will kept static.



cont.

$$H(S) = \frac{SL}{R + SL + \frac{1}{Sc}}$$

$$SL + \frac{1}{Sc} = 0$$

$$H(S) = \frac{S^2 LC}{S^2 LC + SCR + 1} \quad \div LC$$

$$= \frac{S^2}{S^2 + S\frac{R}{L} + \frac{1}{LC}}$$

$$= \frac{S^2}{S^2 + S\frac{\omega_0}{Q} + \omega_0^2}$$

$$\omega_0 = \frac{1}{\sqrt{LC}} \qquad (4)$$

$$\text{Band width } = \beta = \frac{R}{L} \qquad (5)$$

$$\text{Quality factor} = Q = \frac{\omega_0}{\beta} \qquad (6)$$

Magnitude and phase

- Replace ALL S by $j\omega$

$$H(S) = \frac{s^2}{S^2 + S\frac{\omega_0}{Q} + \omega_0^2}$$

$$T(j\omega) = \frac{(j\omega)^2}{(j\omega)^2 + j\omega\frac{\omega_0}{Q} + \omega_0^2}$$

$$|T(j\omega)| = \frac{-\omega^2}{\sqrt{(\omega_0^2 - \omega^2)^2 + (\omega\frac{\omega_0}{Q})^2}}$$

$$\phi(j\omega) = 180 - \tan^{-1}\frac{\omega\frac{\omega_0}{Q}}{\omega_0^2 - \omega^2}$$

Frequncy response (FS) graph

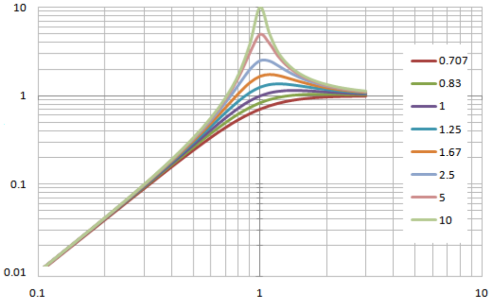
$$|T(j\omega)| = \frac{\omega^2}{\sqrt{(\omega_0^2 - \omega^2)^2 + (\omega\frac{\omega_0}{Q})^2}}$$

1. @ $\omega = zero \Rightarrow$
   $|T(j\omega)| = 0$
2. @ $\omega = \omega_0 \Rightarrow$
   $|T(j\omega)| = \frac{\omega_0^2}{\sqrt{(\frac{\omega_0^2}{Q})^2}} = Q$

$$\phi(j\omega) = 180 - \tan^{-1}\frac{\omega\frac{\omega_0}{Q}}{\omega_0^2 - \omega^2}$$

1. @ $\omega = zero \Rightarrow \phi = 180$
2. @ $\omega = \omega_0 \Rightarrow \phi = 90$
3. @ $\omega = \infty \Rightarrow \phi = 0$

Effect of Q (Magnitude)

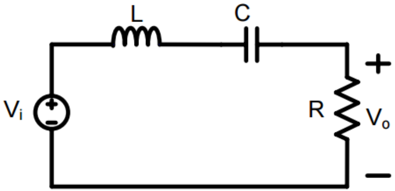


Hint: at $Q = 0.707$ (Flat magnitude) $\omega_0 = \omega_c$

Example

- Find the transfer function.
- compute magnitude, phase.

The BPF cct.:



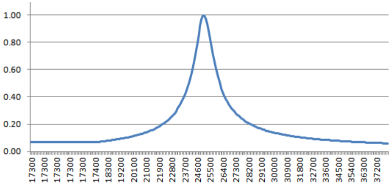
cont.

The BPF cct.:

$$H(S) = \frac{R}{R + SL + \frac{1}{Sc}}$$

$$SL + \frac{1}{Sc} = 0 \qquad \text{purely real}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$



Max. and Min. Frequencies

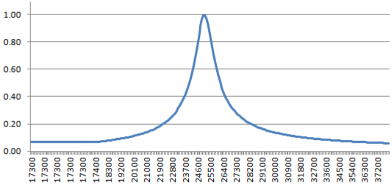
$$H(S) = \frac{R}{R + SL + \frac{1}{Sc}} \times Sc$$

$$= \frac{SCR}{S^2LC + SCR + 1} \div LC$$

$$= \frac{S\frac{R}{L}}{S^2 + S\frac{R}{L} + \frac{1}{LC}}$$

$$= \frac{S\frac{\omega_0}{Q}}{S^2 + S\frac{\omega_0}{Q} + \omega_0^2}$$

The BPF cct.:



$$\phi(j\omega) = 90 - \tan^{-1} \frac{\omega\frac{\omega_0}{Q}}{\omega_0^2 - \omega^2}$$

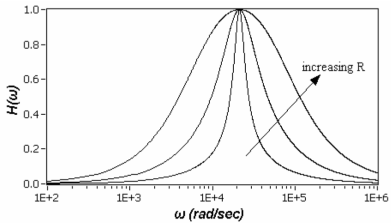
Max. and Min. Frequencies

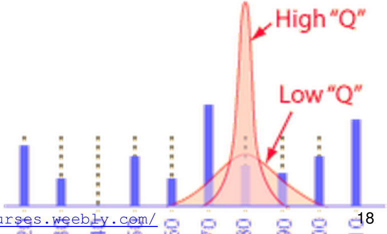
$$\omega_1 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

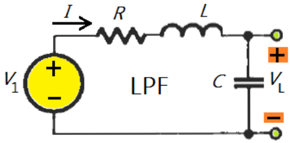
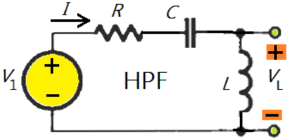
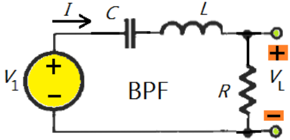
$$\omega_2 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\text{Band width } = \beta = \frac{R}{L}$$

$$\text{Quality factor} = Q = \frac{\omega_0}{\beta}$$





	$H(S)$	$\phi(j\omega)$
 	$= \dfrac{\omega_0^2}{S^2 + S\frac{\omega_0}{Q} + \omega_0^2}$	$= -\tan^{-1} \dfrac{\omega \frac{\omega_0}{Q}}{\omega_0^2 - \omega^2}$
 	$= \dfrac{s^2}{S^2 + S\frac{\omega_0}{Q} + \omega_0^2}$	$= 180 - \tan^{-1} \dfrac{\omega \frac{\omega_0}{Q}}{\omega_0^2 - \omega^2}$
 	$= \dfrac{S\frac{\omega_0}{Q}}{S^2 + S\frac{\omega_0}{Q} + \omega_0^2}$	$= 90 - \tan^{-1} \dfrac{\omega \frac{\omega_0}{Q}}{\omega_0^2 - \omega^2}$