Oscillators

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Shaping Circuits (EEC 242), 2015

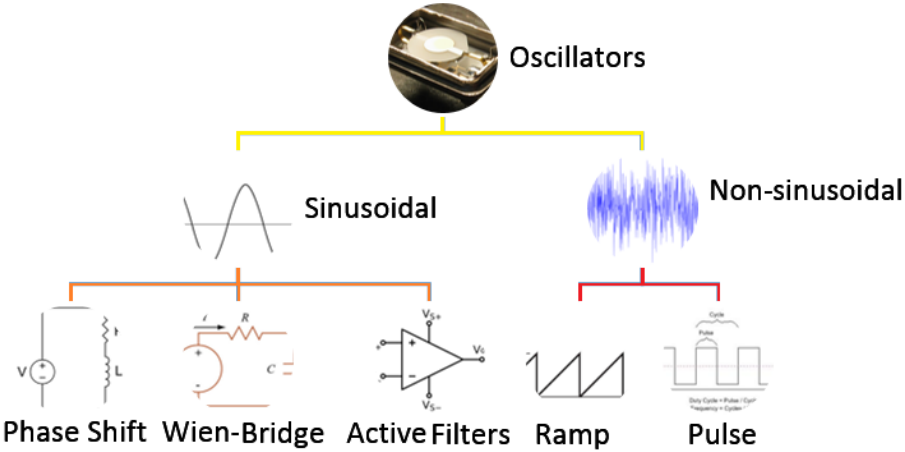
Outline

1. **Basic Principles of Oscillator**

2. **Sinusoidal Oscillators**
   - Phase Shift Oscillator
   - Wien-Bridge Oscillator
   - Active Filter Oscillators
   - LC tuned circuits
   - Quartz Crystals

3. **Non-Sinusoidal Oscillators**
   - Square Oscillator

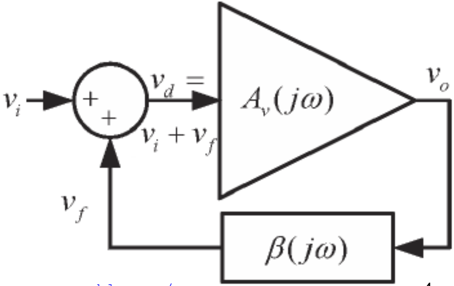
Introduction



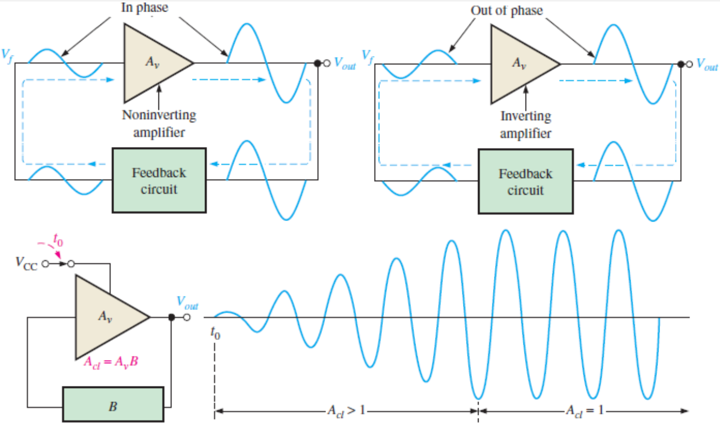
Oscillation Conditions(Barkhausen Criterion)

$$v_d = v_i + v_f$$
$$v_d = v_i + \beta v_o$$
$$V_o = AV_d$$
$$V_o = Av_i + A\beta V_o$$
$$\frac{V_o}{v_i} = \frac{A}{1 - A\beta}$$ with no input
$$\frac{V_o}{v_i} = \infty \Rightarrow 1 - A\beta = 0$$
$$\Rightarrow \boxed{A\beta = 1}$$ loop gain

$$|\beta(j\omega)A(j\omega)| = 1 \qquad (1)$$
$$\angle(\beta(j\omega)A(j\omega)) = 0^\circ = \pm n360^\circ \qquad (2)$$

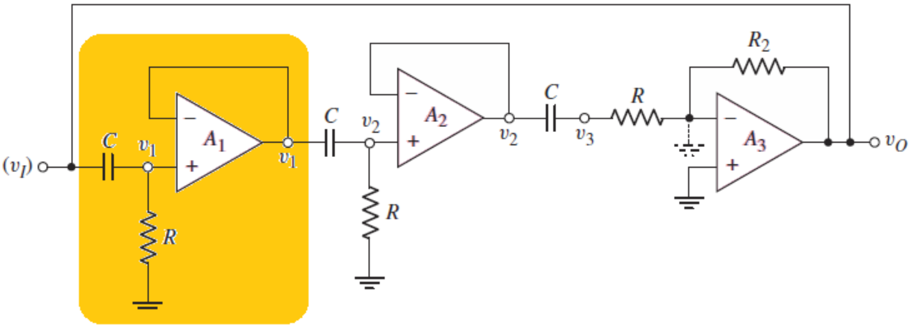


Loop Phase Shift



For oscillation to begin, the voltage gain around the positive feedback loop must be >1 so that the amplitude of the output can build up to a desired level. The gain must then <1 so that the output stays at the desired level and oscillation is sustained. Initially, a small positive feedback voltage develops from thermally produced noise in the resistors/other components/from power supply turn-on transients.

Introduction



$$\frac{v_1}{v_I} = \frac{R}{R + \frac{1}{sc}} = \frac{sRc}{sRC + 1}$$

RC networks are identical

$$\frac{v_3}{v_I} = \left(\frac{sRc}{sRC + 1}\right)^3 = \beta$$

$$A = \frac{v_o}{v_3} = \frac{-R_2}{R}$$

Phase Shift Oscillator: Analysis

Apply Barkhuasen conditions:

1. unity gain $A\beta = 1$

$$|T(j\omega)| = A\beta = \frac{(\frac{R_2}{R})j\omega Rc(\omega Rc)^2}{[1-3(\omega Rc)^2] + j\omega Rc[3-(\omega Rc)^2]} = 1$$

$\therefore$ real=real & img=img

$$\therefore 1 - 3(\omega Rc)^2 = 0 \Rightarrow \omega_o = \frac{1}{\sqrt{3}Rc}$$

$$(\frac{R_2}{R})j\omega Rc(\omega Rc)^2 = j\omega Rc[3-(\omega Rc)^2]$$

$$(\frac{R_2}{R})(\omega Rc)^2 = [3-(\omega Rc)^2]$$

$$(\omega Rc)^2\frac{R_2}{R} = 3 \qquad \text{by using } \omega_o$$

$$R_2 = 8R$$

Phase Shift Oscillator: Analysis 2

Apply Barkhuasen conditions:

2. in Phase $\angle A\beta = 0$

$$\angle T(j\omega) = \angle A\beta = \angle \frac{(\frac{R_2}{R}) j\omega Rc(\omega Rc)^2}{[1-3(\omega Rc)^2] + j\omega Rc[3-(\omega Rc)^2]}$$

$$= tan^{-1}(\frac{\omega Rc(\omega Rc)^2}{0}) - tan^{-1}(\frac{\omega Rc[3-(\omega Rc)^2]}{[1-3(\omega Rc)^2]})$$

$$= 90 - tan^{-1}(\frac{\omega Rc[3-(\omega Rc)^2]}{[1-3(\omega Rc)^2]})$$

to keep phase =0 $\Rightarrow [1-3(\omega Rc)^2 = 0]$

$$1-3(\omega Rc)^2 = 0 \Rightarrow \omega_o = \frac{1}{\sqrt{3}Rc}$$

Phase Shift Oscillator: Example

Example 1

Determine the oscillation frequency and required amplifier gain for a phase-shift oscillator. parameters $C = 0.1\mu F$ and $R = 1k\Omega$.

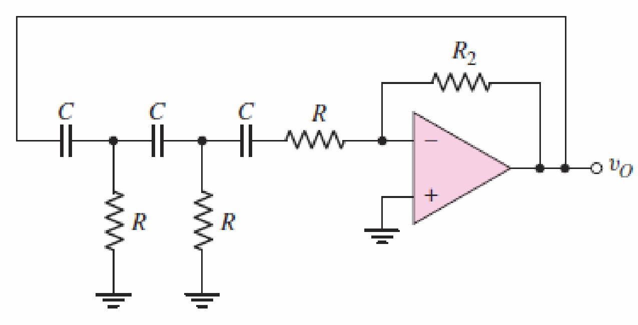
$$f_o = \frac{1}{2\pi\sqrt{3}Rc} = \frac{1}{2\pi\sqrt{3}0.1\mu 1k} = 919Hz$$

$$R_2 = 8R = 8 * 1k = 8K\Omega$$

Example 2

Design the phase-shift oscillator to oscillate at $fo = 22.5kHz$. The minimum resistance to be used is $10k\Omega$.

Phase Shift Oscillator



$$\omega_o = \frac{1}{\sqrt{6}Rc}$$

$$R_2 = 29R$$

Introduction

$$\frac{v_I}{v_o} = \frac{Z_p}{Z_p + Z_s}$$

$$V_o = (1 + \frac{R_2}{R_1}) v_y$$

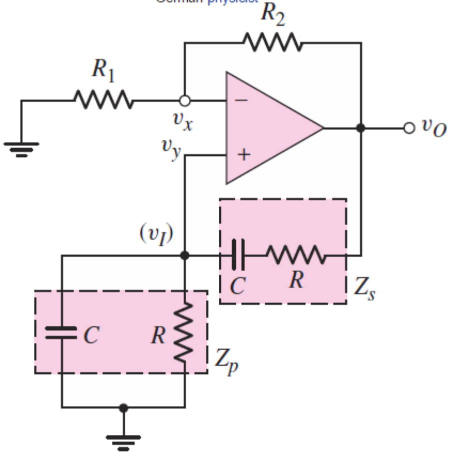
$$\frac{v_o}{v_I} = (1 + \frac{R_2}{R_1}) \frac{Z_p}{Z_p + Z_s}$$

$$Z_p = \frac{R}{1 + ScR}$$

$$Z_s = \frac{1 + ScR}{Sc}$$

$$\boxed{T(S) = (1 + \frac{R_2}{R_1}) \frac{1}{3 + SRc + \frac{1}{SRc}}}$$

Max Wien
1866 – 1938
German physicist



Wien-Bridge Oscillator: Analysis

Apply Barkhuasen conditions:

1. unity gain $A\beta = 1$

$$|T(j\omega)| = A\beta = T(S) = (1 + \frac{R_2}{R_1})\frac{1 + j0}{3 + SRc + \frac{1}{SRc}} = 1$$

$$\therefore \text{real=real \& img=img}$$

$$\therefore SRc + \frac{1}{SRc} = 0 \Rightarrow \omega_o = \frac{1}{Rc}$$

$$(\frac{R_2}{R})(\frac{1}{3}) = 1$$

$$R_2 = 2R$$

Wien-Bridge Oscillator: Analysis 2

Apply Barkhuasen conditions:

2. in Phase $\angle A\beta = 0$

by using ω_o

$$\angle T(j\omega) = \angle A\beta = \angle(1 + \frac{R_2}{R_1})\frac{1 + j0}{3 + SRc + \frac{1}{SRc}}$$

$$= tan^{-1}(\frac{0}{\frac{R_2}{R_1}}) - tan^{-1}(\frac{0}{3}) = 0$$

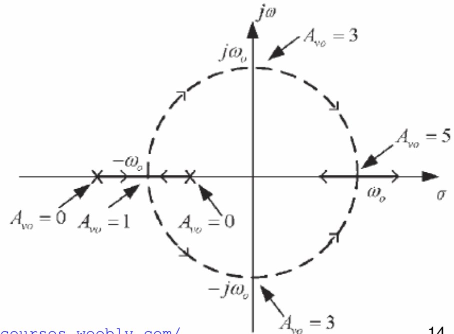
Wien-Bridge Oscillator: Root locus

$$T(j\omega) = 1 - A\beta = T(S) = 1 - \frac{A}{3 + SRc + \frac{1}{SRc}}$$

$$S^2 + (3 - A)\omega_o S + \omega_o^2 = 0$$

$$S = \frac{-(3 - A)\omega_o \pm \sqrt{((3 - A)\omega_o)^2 - 4\omega_o^2}}{2}$$

$$A = \begin{cases} 0 \longrightarrow 1 & \text{real} \\ 1 < A < 3 & \text{Complex} \\ A = 3 & \text{imagenary} \end{cases}$$



Wien-bridge Oscillator: Example

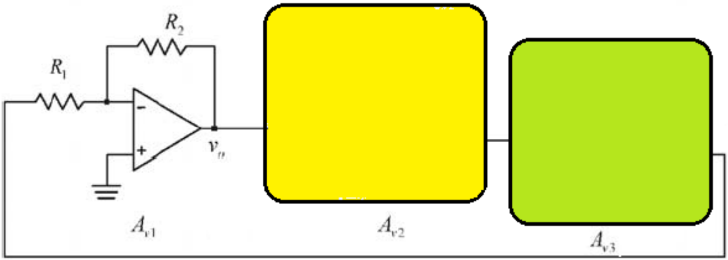
Example 1

Determine the oscillation frequency and required amplifier gain for a Wien-bridge oscillator. parameters $C = 0.02\mu F$ and $R = 10k\Omega$.

Example 2

Design the Wien-bridge oscillator to oscillate at $fo = 20kHz$. The minimum resistance to be used is $10k\Omega$.

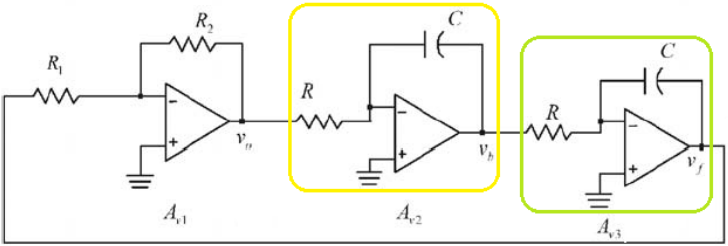
Introduction



$$\angle T(S)|_{\text{inverting op amp}} = 180$$

Therefore the yellow and green block must have phases equal ??
The overall gain $A_{v1}A_{v2}A_{v3} = ?$
Could you suggest circuits that realize the previous conditions!!

Active Filter Oscillators



To get -90° phase, we need $\frac{1}{s}$ which represent LPF. $180-90-90=0$
The overall gain $A_{v1}A_{v2}A_{v3}=?$

$$\beta A_{v1}A_{v2}A_{v3}=1$$

$$-\frac{R_2}{R_1}\left(\frac{1}{sRC}\right)^2=1 \Rightarrow \omega_o=\frac{1}{RC}\sqrt{\frac{R_2}{R_1}}$$

Active Filter Oscillator: Example

Example 1

Design the Active Filter oscillator to oscillate at $fo = 20kHz$. The minimum resistance to be used is $10k\Omega$.

Analysis 1

By using KCL(Colpitts):

$$\frac{V_o}{\frac{1}{SC_1}} + \frac{V_o}{R} + g_m V_{gs} + \frac{V_o}{SL + \frac{1}{SC_2}} = 0$$

$$V_{gs} = \frac{V_o \frac{1}{SC_2}}{SL + \frac{1}{SC_2}} = \frac{V_o}{S^2 LC_2 + 1}$$

$$V_o[SC_1 + \frac{1}{R} + \frac{g_m}{S^2 LC_2 + 1} + \frac{1}{SL + \frac{1}{SC_2}}] = 0$$

If oscillation has started, then $Vo \neq 0$

$$\therefore SC_1 + \frac{1}{R} + \frac{g_m}{S^2 LC_2 + 1} + \frac{SC_2}{S^2 LC_2 + 1} = 0$$

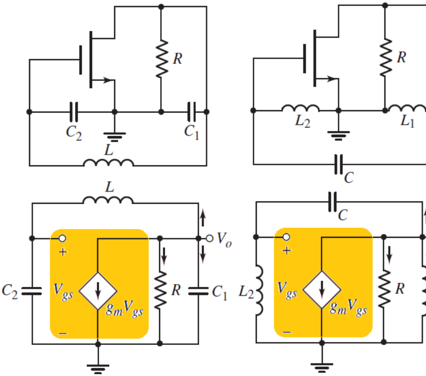
$$(S^2 LC_2 + 1)(SC_1 + \frac{1}{R}) + g_m + SC_2 = 0$$

$$S^3 LC_1 C_2 + S^2 \frac{LC_2}{R} + S(C_1 + C_2) + (g_m + \frac{1}{R}) = 0$$

$$[g_m + \frac{1}{R} - \omega^2 \frac{LC_2}{R}] + j\omega[(C_1 + C_2) - \omega^2 LC_1 C_2] = 0$$

Colpitts Oscillator
Edwin Henry Colpitts
(1872 – 1949), Canada

Hartley Oscillator
Ralph Vinton Lyon Hartley
(1888 –1970) United States



Analysis 2

By using KCL(Colpitts):

$$[g_m + \frac{1}{R} - \omega^2 \frac{LC_2}{R}] + j\omega[(C_1 + C_2) - \omega^2 LC_1 C_2] = 0$$

$$\omega[(C_1 + C_2) - \omega^2 LC_1 C_2] = 0 \qquad (3)$$

$$g_m + \frac{1}{R} - \omega^2 \frac{LC_2}{R} = 0 \qquad (4)$$

From 3:

$$\omega[(C_1 + C_2) - \omega^2 LC_1 C_2] = 0$$

$$\omega^2 = \frac{1}{L\frac{C_1 C_2}{C_1+C_2}} = \frac{1}{LC_T} \Rightarrow \omega = \frac{1}{\sqrt{LC_T}}$$

$$\because C_T = C_1 || C_2$$

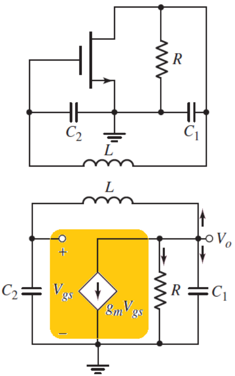
From 4:

$$\frac{C_2}{C_1} = g_m R$$

Colpitts Oscillator

Edwin Henry Colpitts

(1872 – 1949), Canada



Hartley Oscillator

Ralph Vinton Lyon Hartley

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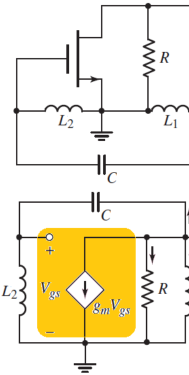
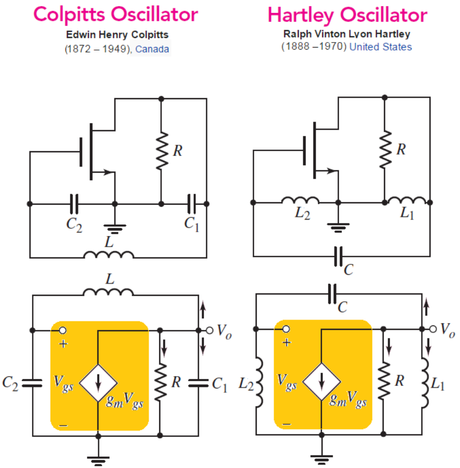


Analysis 3

By using KCL(Hartley):

$$\omega = \frac{1}{\sqrt{CL_T}}$$

$$\because L_T = L_1 + L_2$$



LC Tuning circuit: Example

Example 1

For the Colpitts oscillator, assume parameters of $L = 1\mu H$, C_1 and $C_2 = 1nF$, and $R = 4k\Omega$. Determine the oscillator frequency and the required value of g_m. (Ans. $f_o = 7.12MHz$, $g_m = 0.25mA/V$)

Piezoelectric Effect

Quartz

Potassium sodium tartrate

Rochelle salt (1675)

Pierre Seignette, France.

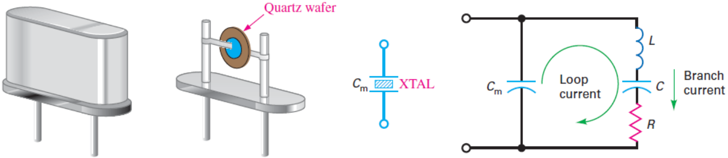
Tourmaline

semi-precious stone

SriLanka.

When an ac voltage is applied across them, they vibrate at the frequency of the applied voltage. Conversely, if they forced to mechanically vibrate, they generate an ac voltage of the same frequency. This phenomenon is called "Piezoelectric".

AC Equivalent Circuit



(a) Typical packaged crystal (b) Basic construction (without case) (c) Symbol (d) Electrical equivalent

Cm (mounting capacitance): presents the two metal plates separated by a dielectric.
L,C,R: represent the motion of the crystal (since L = the electrical equivalent of the crystal mass, C= the crystal stiffness or elasticity, and R = the heat losses due to mechanical friction in the crystal).

$$Z(s) = \frac{S^2 + S\frac{R}{L} + \frac{1}{LC}}{Sc_m[S^2 + S\frac{R}{L} + (1 + \frac{C}{C_m})\frac{1}{LC}]}$$

Resonant Frequencies

$$Z(s) = \frac{S^2 + S\frac{R}{L} + \frac{1}{LC}}{Sc_m[S^2 + S\frac{R}{L} + (1 + \frac{C}{C_m})\frac{1}{LC}]}$$

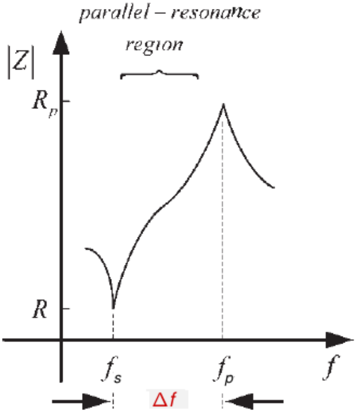
$$= \frac{S^2 + S\frac{\omega_s}{Q} + \omega_s^2}{Sc_m[S^2 + S\frac{\omega_s}{Q} + \omega_p^2]}$$

It is clear there are two frequencies (series resonance = ω_s^2, parallel resonance=ω_s^2)

$$\omega_s = \frac{1}{\sqrt{LC}}$$

$$\omega_p = \frac{1}{\sqrt{LC_T}} \qquad \because C_T = C||C_m$$

$$Q = \frac{\omega_s L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}}$$



Pulling Range Δf

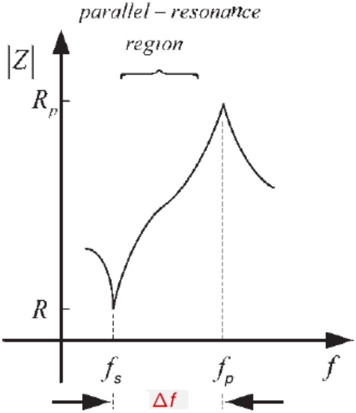
$$\Delta f = f_p - f_s \approx f_s \left(\frac{C}{2C_m} \right)$$

Two figures of merit used in Data-sheets r and M: The parameter r is defined as

$$r = \frac{C_m}{C}$$

$$M = Qr = \frac{1}{\omega_s R C_m}$$

It provides a measure of the separation between f_s and f_a



Crystal Oscillator: Example

Example 1

Consider a 2-MHz series resonant crystal described by $L = 0.528H$, $C = 0.011993pF$, $R = 100\Omega$, and $C_m = 4pF$. Calculate the series-resonance frequency f_s , the parallel-resonant frequency f_p , and the crystal Q. Also, calculate the figure of merits r and M.

(Ans. $f_s = 2.000042MHz$, $f_p = 2.003051MHz$, $Q = 66,350$, $\Delta f = 3kHz$, r=333.5 , and M=199)

Types of Crystal Oscillators

- **XO**: A crystal oscillator is a quartz crystal packaged with its oscillator circuitry.
- **VCXO**: A voltage-controlled crystal oscillator is an XO but optimized for external frequency control by way of a dc input (application: PLL).
- **TCXO**: A temperature-compensated crystal oscillator is one that incorporates circuitry to compensate for the frequency variations that accompany temperature variations (application: Cell phone).
- **OCXO**: An oven-compensated crystal oscillator puts the crystal and sometimes the whole oscillator circuit in a small oven. A dc heating element in a feedback loop keeps the temperature virtually constant, giving a very precise and stable output frequency.

Frequency Accuracy

Measures of how close the crystal frequency is to the desired value. It is often expressed as a percentage deviation from the specifi ed frequency or in parts per million (ppm).

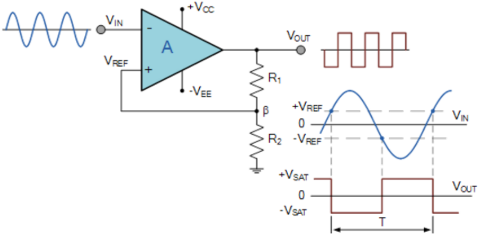
Example

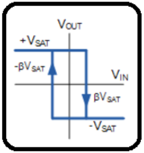
A 10MHz Crystal oscillator has an accuracy of 100 ppm. calculate the tolerance of this oscillator:

$$\frac{100}{1,000,000} \times 10,000,000 = 1000Hz$$

Relaxation Oscillators (Introduction)

Op-amp Schmitt Comparator:





$$V_{Ref} = V^+ = V_o \frac{R_1}{R_1 + R_2} = \beta V_o$$

$$V_o = \begin{cases} V_s^+ = V_{DD} - 1 & V_{in} > V_{Ref} \\ V_s^- = V_{SS} + 1 & V_{in} < V_{Ref} \end{cases}$$

Relaxation Oscillators Using Operational Amplifiers

To find oscillation frequency:

$$v_c = A + Be^{-\frac{t-t_a}{RC}}$$

$$v_c|_{(t=t_a)} = -v_{Ref} = A + B$$

$$v_c|_{(t=\infty)} = v_{Ref} = A$$

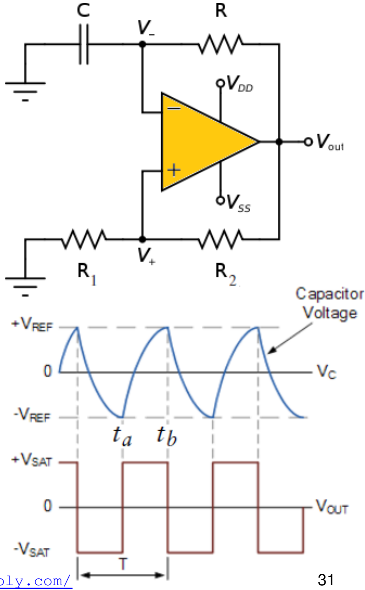
$$\because V_{Ref} = V_o \frac{R_1}{R_1 + R_2}$$

$$= 0.5V_o = 0.5V_s^- \qquad R_1 = R_2$$

$$\therefore -0.5V_s^- = A + B \quad \& V_s = A$$

$$\therefore B = -\frac{3}{2}V_s$$

$$v_c = V_s(1 - \frac{3}{2}e^{-\frac{t-t_a}{RC}})$$



Relaxation Oscillators Using Operational Amplifiers

To find oscillation frequency:

$$v_c = V_s(1 - \frac{3}{2}e^{-\frac{t-t_a}{RC}})$$

$$0.5V_s = V_s(1 - \frac{3}{2}e^{-\frac{t_b-t_a}{RC}})$$

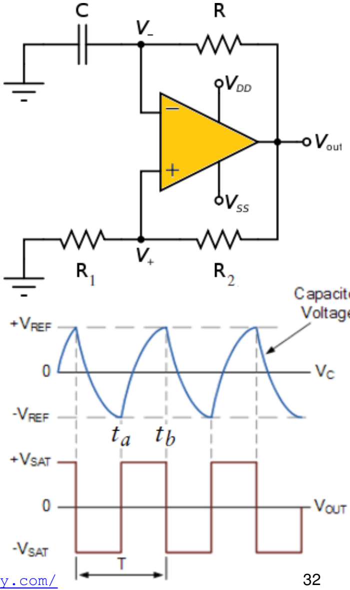
$$0.5 = -\frac{3}{2}e^{-\frac{t_b-t_a}{RC}}$$

$$ln(3) = \frac{t_b - t_a}{RC}$$

$$(t_b - t_a) = ln(3)RC$$

$$\boxed{T = 2(t_b - t_a) = 2.197RC \approx 2.2RC}$$

$$\boxed{T = 2RC\ ln(1 + \frac{2R_1}{R_2})}$$



Example

Design the relaxation oscillator to oscillate at 8 kHz.
Solution:
Let $R_1 = R_2 = 10k\Omega$ and $C = 10nF$. $R = \frac{1}{2.2Cf} = 5.68k\Omega$