Subject: Shaping Circuits /(EEC 242)(G1)
Examiner : Dr. Mustafa M. Shiple

Term: Sep - Jan 2015/16
Exam Time:60 min

ANSWER THE FOLLOWING QUESTIONS:

1. A Second-order passive lowpass filter is designed with: [10 marks] [A_a, C_p]

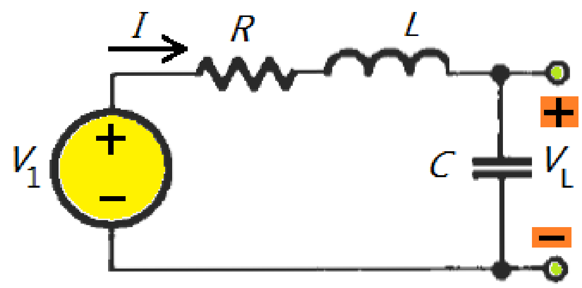
   $R = 120\Omega$,$L = 16mH$, and $C = 0.05\mu F$.

   (a) Drive the transfer function.

   (b) Calculate ω_o and Q.

   (c) Redesign the circuit to raise its quality factor to 12 without changing ω_o. Draw the wiring diagram of the circuit.

Solution: Q1.(a)



$$H(S) = \frac{\frac{1}{Sc}}{R + SL + \frac{1}{Sc}} \qquad \times SC$$

$$H(S) = \frac{1}{S^2LC + SCR + 1} \qquad \div LC$$

$$= \frac{\frac{1}{LC}}{S^2 + S\frac{R}{L} + \frac{1}{LC}}$$

$$= \frac{\omega_0^2}{S^2 + S\frac{\omega_0}{Q} + \omega_0^2}$$

$$\text{Quality factor} = Q = \frac{\omega_0}{\beta} \quad \omega_o = \frac{1}{\sqrt{LC}} \quad \beta = \frac{R}{L}$$

(b)

$$\omega_o = \frac{1}{\sqrt{LC}} = \frac{1}{\sqrt{16m \times 0.05\mu}} = 35.35\ krad/sec$$

$$Q = \frac{\omega_0}{\beta} = \frac{1}{\sqrt{LC}} \times \frac{L}{R} = \frac{1}{R}\sqrt{\frac{L}{C}} = 4.71$$

(c)

$$\because \omega_o \text{ dependes on } L, C$$

$$\because \frac{\omega_o}{Q} = \frac{R}{L} \Rightarrow \text{the only parameter i could change without affecting } \omega_o \text{ is } R$$

$$\therefore R = \frac{L\omega_o}{Q} = 47.13\Omega$$

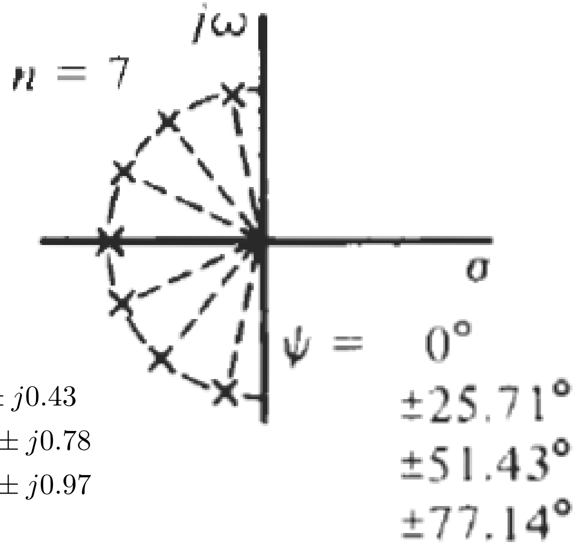
[Total Marks is 30]

2. For 7^{th} order Butterworth. Find the pole locations. [5 marks] $[B_a, A_d, A_q]$

Solution:

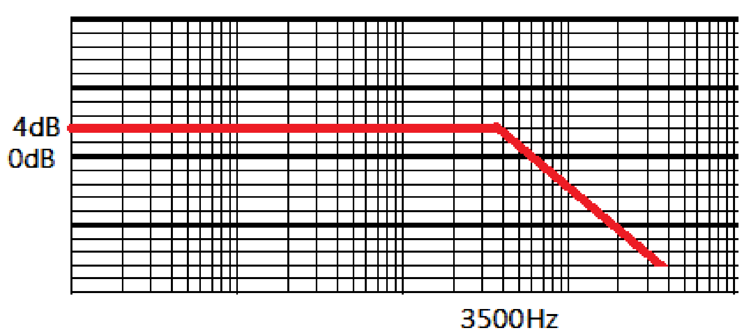
$$\psi = \frac{180}{7} = 25.71^\circ$$

$$B = (S+1)(S^2 + 2\cos 25.71 S + 1)$$
$$(S^2 + 2\cos 51.43 S + 1)$$
$$(S^2 + 2\cos 77.14 S + 1)$$



$$P_1 = -1$$
$$P_{2,3} = \cos 25.71 \pm \sin 25.71 = -0.9 \pm j0.43$$
$$P_{4,5} = \cos 51.43 \pm \sin 51.43 = -0.62 \pm j0.78$$
$$P_{6,7} = \cos 77.14 \pm \sin 77.14 = -0.22 \pm j0.97$$

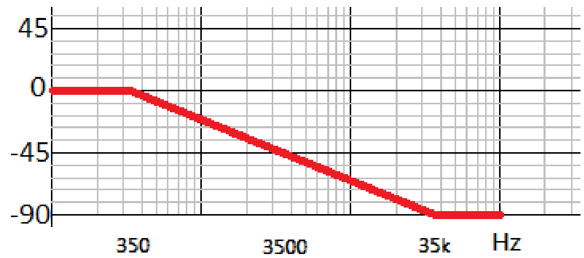
3. Consider the Bode asymptotic plot in the next Figure. [15 marks] $[B_a, A_d, A_q]$
   (a) Determine $T(s)$ and plot the phase of $T(s)$.
   (b) Drive an expression for transfer function of Sallen-key.
   (c) Find the Sallen-key circuit that will realize the given specifications. Find the proper values of the circuit components.
   (d) Calculate the error percentage of Q if the R_F increased 10%.



Solution:
From the bode plot:
$Gain = 4dB, \quad maximum\ flat \Rightarrow Q = 0.707, \quad w_o = 2\pi \times 3500 = 21.98k\ rad/sec$
(a)

$$k = 10^{\frac{4}{20}} = 1.58$$

$$T(S) = \frac{1.58}{((\frac{S}{21.98k}) + 1)}$$



(b)

$$V^-(1 + \frac{R_F}{R}) = V^- K = V_{out} V_x \quad :$$

$$\frac{V_{in} - V_x}{Z_1} = \frac{V_x - V^-}{Z_2} + \frac{V_x - V_{out}}{Z_3}$$

$$\frac{V_x - V^-}{Z_2} = \frac{V^-}{Z_4}$$

$$\therefore V_x = V^- \left(\frac{Z_2}{Z_4} + 1\right)$$

$$V_{in} - V_x = V_x\frac{Z_1}{Z_2} - V^-\frac{Z_1}{Z_2} + V_x\frac{Z_1}{Z_3} - V_{out}\frac{Z_1}{Z_3}$$

$$V_{in} = V_x(\frac{Z_1}{Z_2} + \frac{Z_1}{Z_3} + 1) - V^-\frac{Z_1}{Z_2} - V_{out}\frac{Z_1}{Z_3}$$

$$V_{in} = \frac{V_{out}}{K}\left(\frac{Z_2}{Z_4} + 1\right)(\frac{Z_1}{Z_2} + \frac{Z_1}{Z_3} + 1) - \frac{V_{out}}{K}\frac{Z_1}{Z_2} - V_{out}\frac{Z_1}{Z_3}$$

$$\frac{V_{in}}{V_{out}} = \frac{1}{K}\left(\frac{Z_1}{Z_4} + \frac{Z_1 Z_2}{Z_4 Z_3} + \frac{Z_2}{Z_4} + \frac{Z_1}{Z_2} + \frac{Z_1}{Z_3} + 1 - \frac{Z_1}{Z_2}\right) - \frac{Z_1}{Z_3}$$

$$= \frac{Z_1Z_3 + Z_1Z_2 + Z_2Z_3 + Z_1Z_4 + Z_3Z_4 - KZ_1Z_4}{KZ_3Z_4}$$

$$\frac{V_{out}}{V_{in}} = \frac{KZ_3Z_4}{Z_1Z_3 + Z_1Z_2 + Z_2Z_3 + Z_1Z_4 + Z_3Z_4 - KZ_1Z_4}$$

(c)

For Low pass filter: $Z_3,\ Z_4 are capacitors$ and $Z_1,\ Z_2 are resistors$

$$\frac{V_{out}}{V_{in}} = \frac{K\frac{1}{SC_3}\frac{1}{SC_4}}{R_1\frac{1}{SC_3} + R_1R_2 + R_2\frac{1}{SC_3} + R_1\frac{1}{SC_4} + \frac{1}{SC_3}\frac{1}{SC_4} - KR_1\frac{1}{SC_4}} \qquad \times S^2C_4C_3$$

$$= \frac{K}{R_1SC_4 + s^2C_3C_4R_1R_2 + R_2SC_4 + R_1SC_3 + 1 - KR_1SC_3} \qquad \div C_3C_4R_1R_2$$

$$= \frac{\frac{K}{C_3C_4R_1R_2}}{s^2 + S\frac{R_1C_4 + R_2C_4 + R_1C_3 - KR_1C_3}{C_3C_4R_1R_2} + \frac{1}{C_3C_4R_1R_2}} \qquad C_3 = C_4 = C$$

$$\boxed{T(S) = \frac{\frac{K}{C^2R_1R_2}}{s^2 + S\frac{R_2 + R_1(2-K)}{CR_1R_2} + \frac{1}{C^2R_1R_2}}}$$

$$\omega_o = \frac{1}{C\sqrt{R_1 R_2}} \quad Q = \frac{\sqrt{R_1 R_2}}{R_2 + R_1(2-K)}$$

it is obvious that Q depends on R_1,R_2, and k which is equal 1.58
$w_o = 3500 rad/sec$ depends on R_1,R_2, and C.
So we will use R_1,R_2 to control Q and C to tune the w_o.

$$Q = \frac{\sqrt{R_1 R_2}}{R_2 + R_1(2-K)}$$

$$0.707 = \frac{\sqrt{R_1 R_2}}{R_2 + 0.42R_1}$$

$$0.707R_2 + 0.3R_1 = \sqrt{R_1 R_2} \qquad let R_1 = 10k\Omega$$

$$0.707R_2 + 3k = 100\sqrt{R_2}$$

$$(0.707R_2 + 3k)^2 = 10000R_2$$

$$0.5R_2^2 - 5859R_2 + 9 \times 10^6 = 0$$

$$R_2 \approx 9900\Omega \text{ or } R_2 \approx 1818\Omega$$

To find C:

$$C = \frac{1}{w_o\sqrt{R_1 R_2}}$$

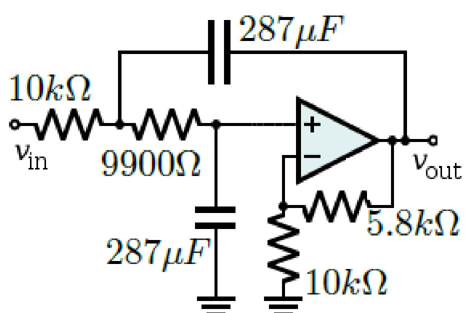
by substituting $R_1 = 10k\Omega, w_o = 3500rad/sec, R_2 = 9900/1818\Omega$

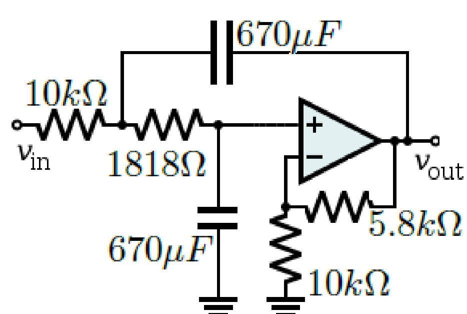
$$\therefore C = 287\mu F \text{ or } 670\mu F$$

To find R_i, R_F

$$R_F = (k-1)R_i \qquad \text{let } k = 1.58, R_i = 10k\Omega$$

$$R_F = 5.8k\Omega$$





(d)

$$R_F = 5800 + 580 = 6380\Omega$$

$$K = 1 + \frac{R_F}{R_i} = 1.638$$

$$Q = \frac{\sqrt{R_1 R_2}}{R_2 + R_1(2-K)}$$

- $@R_2 = 9800 \Rightarrow Q = 0.735 \Rightarrow error = 4\%$
- $@R_2 = 1818 \Rightarrow Q = 0.784 \Rightarrow error = 10\%$