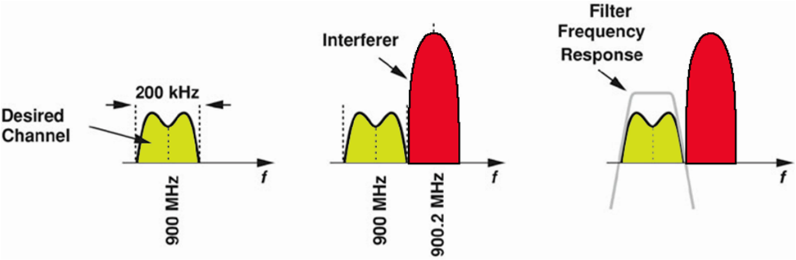
Introduction To Analog Filters

Dr. M. Shiple

Shaping Circuits (EEC 242), 2015

Outline

Cell phone

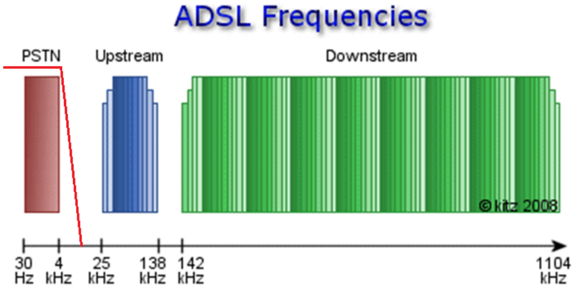


Application: Cellphone
Center frequency: 900 MHz
Bandwidth: 200 KHz

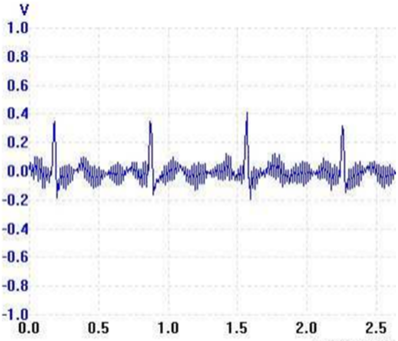
Adjacent interference

Use a filter to remove interference

ADSL Splitter



Heart Rate

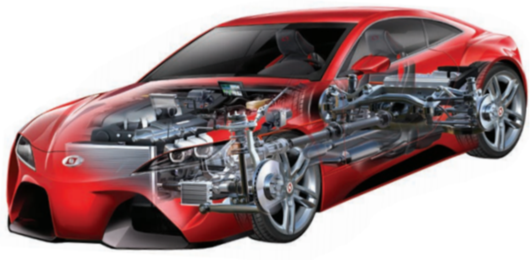


Automotive Applications

Current Sense Amplifiers

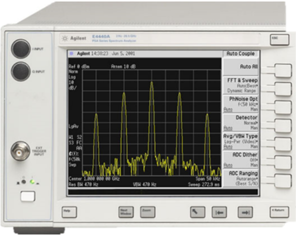
Applications:

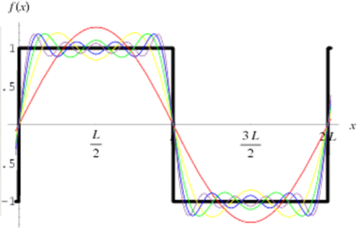
- H-Bridge Motor Control
- Solenoid Current Sense
- PWM Control Loops
- Hydraulic Controls
- Lamp Monitoring
- Glow Plug Control
- Load Monitoring
- HEV/EV Battery Management Systems
- 12V / 24V Battery Monitoring
- High Voltage Data Acquisition



Part Number	Current Direction	Common Mode Voltage (V)	Response Time (µs)	V_{OS} Max (µV)	PSRR Min (dB)	Max Temperature Range	Comments
LT1787	Bidirectional	2.5 to 65	10	150	100	–40°C to 125°C	Buffered Output; Simple Input Filtering
LT1999	Bidirectional	–5 to 80	2.5	750	80	–55°C to 150°C	High Speed AC Monitor
LT6100	Unidirectional	4.1 to 48	40	300	95	–40°C to 125°C	Buffered Output with 5 Gain Settings

Fourier Series



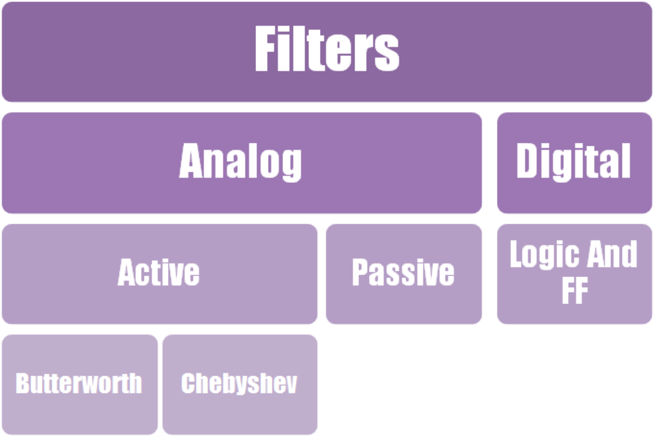


$$a_o = \frac{2}{T}\int_a^{a+T} f(t)dt \tag{1}$$

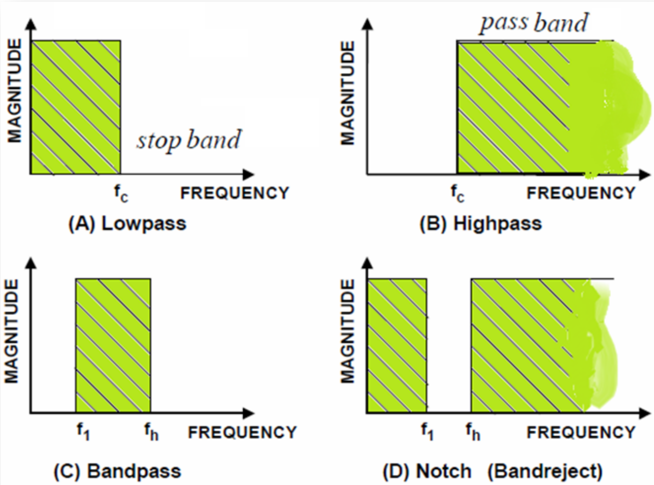
$$a_n = \frac{2}{T}\int_a^{a+T} f(t)\cos(kwt)dt, \qquad k \geq 1 \tag{2}$$

$$a_n = \frac{2}{T}\int_a^{a+T} f(t)\sin(kwt)dt, \qquad k \geq 1 \tag{3}$$

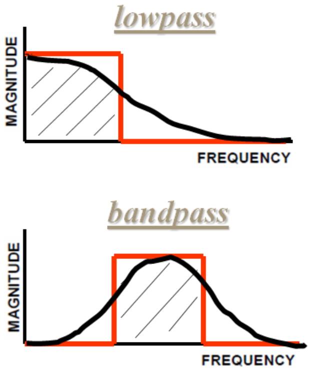
Hierarchy of Filters



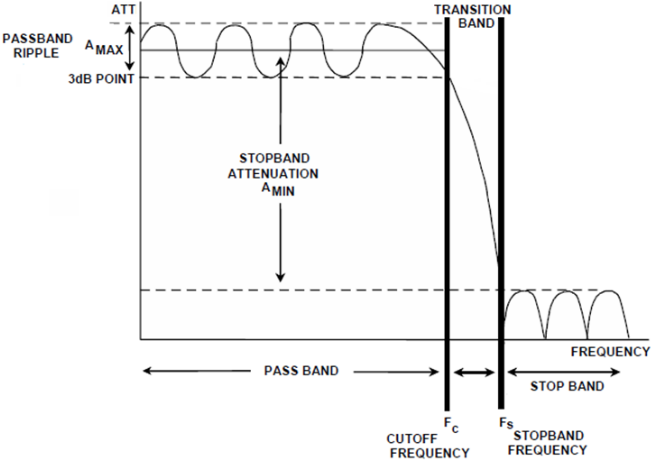
"Ideal Filters" Frequency domain



"Realistic Filters" Frequency domain



Key Filter Parameters



General form

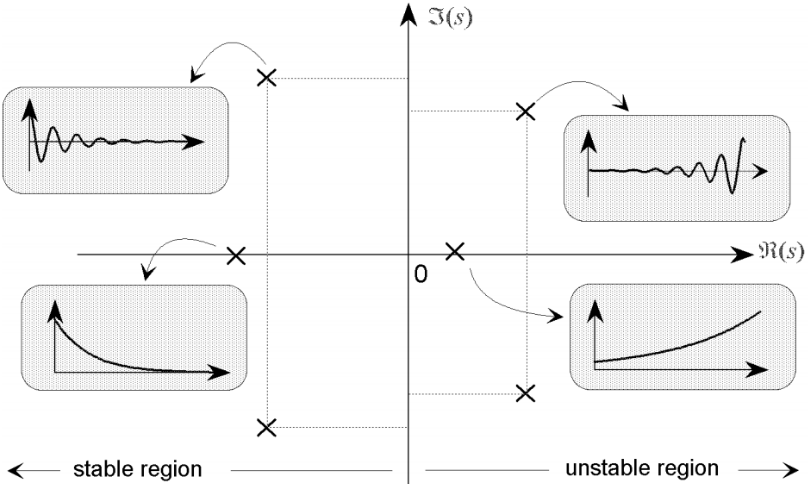
$$T(s) = \frac{N(s)}{D(s)} = \frac{b_m s^m + b_{m-1} s^{m-1} + \cdots + b_1 s + b_0}{a_n s^n + a_{n-1} s^{n-1} + \cdots + a_1 s + a_0}$$

$$T(s) = \frac{N(s)}{D(s)} = \frac{b_1 s + b_0}{a_1 s + a_0}$$ First Order

Conditions

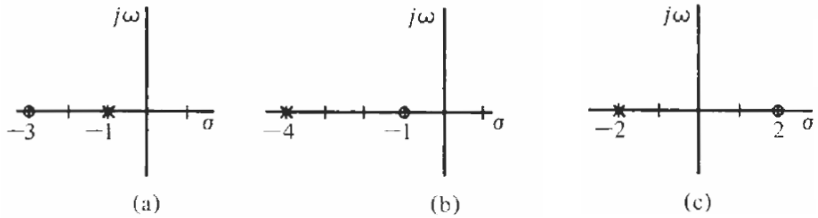
- a_i and b_i are real numbers.
- a_i could be positive, negative, and zero.
- b_i could be positive.

Examples

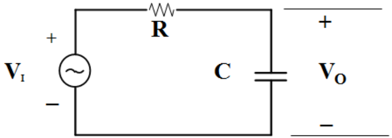


Examples

$$T_a(s) = K_a \frac{s+3}{s+1} \qquad T_b(s) = K_b \frac{s+1}{s+4} \qquad T_c(s) = K_c \frac{s-2}{s+2}$$



Examples



- Find the transfer function.
- compute magnitude, phase, pole, and zero , assume $R = 12k\Omega$ and $C = 100nf$.

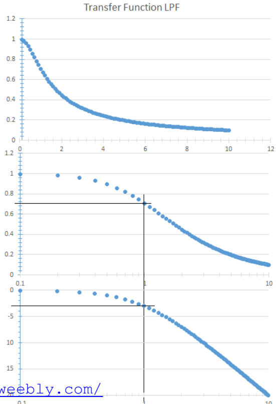
Solution

$|T(S)|$

Magnitude

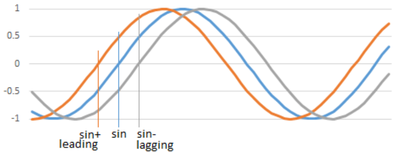
$$\alpha = 20log_{10}|T(S)|$$

$$f_c = \frac{1}{2\pi Rc} = \frac{1}{2 \times 3.14 \times 12k \times 100n} = 132.6Hz$$

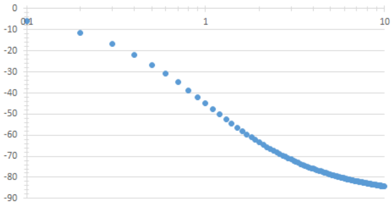


Low Pass Filter

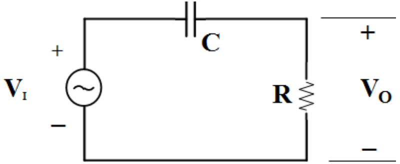
$< \theta(\omega)$



Phase



Examples



- Find the transfer function.
- compute magnitude, phase, pole, and zero , assume $R = 12k\Omega$ and $C = 100nf$.

Solution

$|T(S)|$

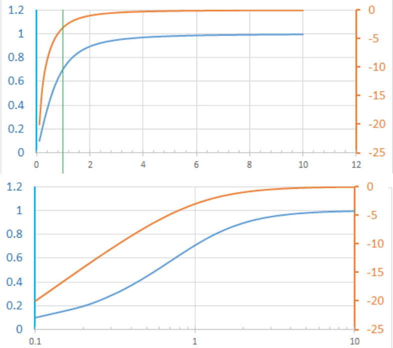
$$\alpha = 20 log_{10}|T(S)|$$

$$f_c = \frac{1}{2\pi Rc}$$

$$= \frac{1}{2 \times 3.14 \times 12k \times 100n}$$

$$= 132.6Hz$$

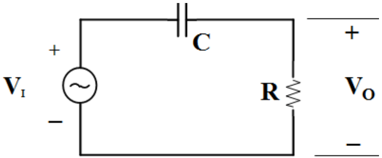
Magnitude



Excr 1:

Design a 1^{st} order HPF with next specifications:

- $|T(0)| = 0.3$.
- $|T(\infty)| = 1$.
- there are a zero @ $f_z = -159$.

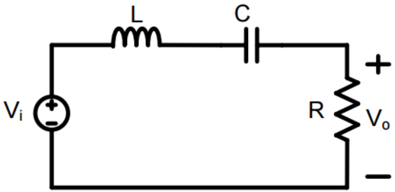


Solution

Example

- Find the transfer function.
- compute magnitude, phase.

The BPF cct.:



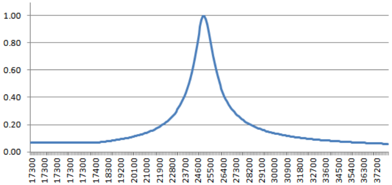
cont.

The BPF cct.:

$$H(S) = \frac{R}{R + SL + \frac{1}{Sc}}$$

$$SL + \frac{1}{Sc} = 0 \qquad \text{purely real}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

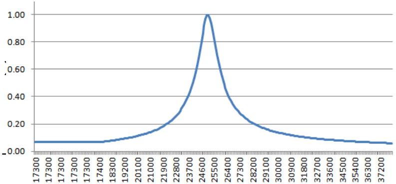


Band Pass Filter

- Therefore at the resonant frequency the impedance seen by the source is purely resistive.
- This implies that at resonance the inductor/capacitor combination acts as a short circuit.
- The current flowing in the system is in phase with the source voltage.

Max. and Min. Frequencies

The BPF cct.:



$$H(S) = \frac{R}{R + SL + \frac{1}{Sc}}$$

$$= \frac{SCR}{S^2 LC + SCR + 1}$$

$$= \frac{S\frac{R}{L}}{S^2 + S\frac{R}{L} + \frac{1}{LC}}$$

$$|T(j\omega)| = \frac{\frac{R\omega}{L}}{\sqrt{(\frac{1}{LC} - \omega^2)^2 + \left(\frac{\omega R}{L}\right)^2}}$$

$$|T(j\omega)| = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{(\frac{\omega L}{R} - \frac{1}{\omega RC})^2 + 1}}$$

$$\pm 1 = \frac{\omega L}{R} - \frac{1}{\omega RC}$$

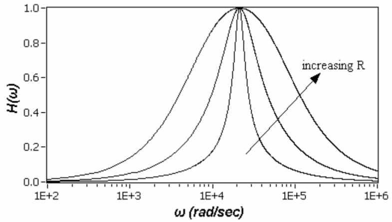
Max. and Min. Frequencies

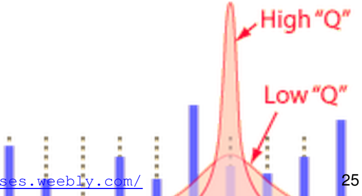
$$\omega_1 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\omega_2 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\text{Band width } = \beta = \frac{R}{L}$$

$$\text{Quality factor} = Q = \frac{\omega_0}{\beta}$$

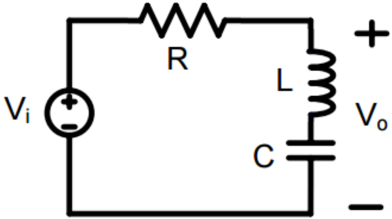




Example

- Find the transfer function.
- compute magnitude, phase.

The BRF cct.:



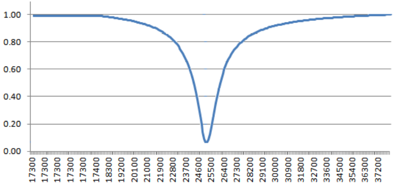
cont.

$$H(S) = \frac{SL + \frac{1}{Sc}}{R + SL + \frac{1}{Sc}}$$

$$SL + \frac{1}{Sc} = 0 \qquad \text{purely real}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

The BRF cct.:

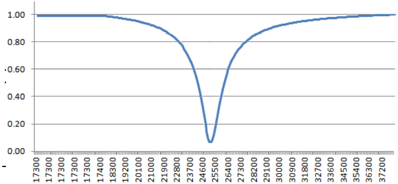


Max. and Min. Frequencies

$$H(S) = \frac{SL + \frac{1}{Sc}}{R + SL + \frac{1}{Sc}} = \frac{S^2LC + 1}{S^2LC + SCR + 1} = \frac{S^2 + \frac{1}{LC}}{S^2 + S\frac{R}{L} + \frac{1}{LC}}$$

$$|T(j\omega)| = \frac{\sqrt{(\frac{1}{LC} - \omega^2)^2}}{\sqrt{(\frac{1}{LC} - \omega^2)^2 + (\frac{\omega R}{L})^2}}$$

The BRF cct.:



$$|T(j\omega)| = \frac{1}{\sqrt{2}} = \frac{1}{\sqrt{(\frac{\frac{\omega L}{R}}{\frac{1}{LC} - \omega^2})^2 + 1}}$$

Max. and Min. Frequencies

$$\omega_1 = \frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

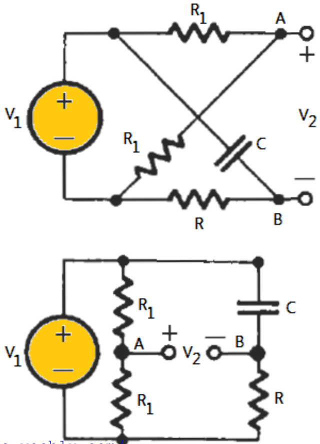
$$\omega_2 = -\frac{R}{2L} + \sqrt{\left(\frac{R}{2L}\right)^2 + \frac{1}{LC}}$$

$$\text{Band width } = \beta = \frac{R}{L}$$

$$\text{Quality factor} = Q = \frac{\omega_0}{\beta}$$

Example

- Find the transfer function.
- compute magnitude, phase.



cont.

$$V_A = V_1 \frac{R_1}{2R_1} = \frac{1}{2}$$

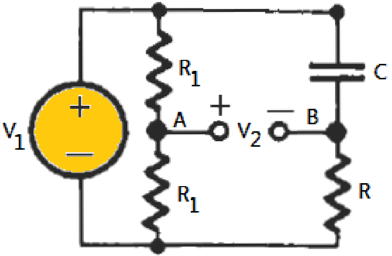
$$V_B = V_1 \frac{SCR}{SCR+1}$$

$$V_2 = V_A - V_B = v_1 \left(\frac{1}{2} - \frac{SCR}{SCR+1} \right)$$

$$\omega_c = \frac{1}{RC}$$

$$H(S) = \frac{1}{2} \times \frac{1-SCR}{1+SCR}$$

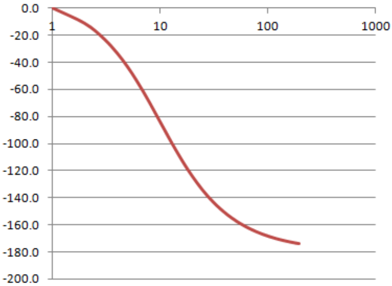
The APF cct.:



Max. and Min. Frequencies

$$|T(j\omega)| = \frac{1}{2} \qquad \text{for all } \omega$$

$$\theta = -2\tan^{-1}\frac{\omega}{\omega_c}$$



Example 3.1:

By using voltage divider:

$$V_o = V_i \frac{\frac{1}{Sc}}{R + \frac{1}{Sc}} \qquad \times SC$$

$$\frac{V_o}{V_i} = \frac{1}{1 + SRc} = \frac{1}{1 + j\omega Rc}$$

To find the pole of TF (Dnum = 0):

$$|T(S)| = \frac{1}{\sqrt{(1)^2 + (\omega Rc)^2}} = \frac{1}{\sqrt{2}}$$

$$\omega_c = \frac{1}{Rc}$$

To find the zero of TF (Num = 0):

zero $= \infty$

To find the magnitude of TF:

$$|T(S)| = \frac{1}{\sqrt{(1)^2 + (\omega Rc)^2}} \quad \text{use } \omega_c$$

$$|T(S)| = \frac{1}{\sqrt{1 + (\frac{\omega}{\omega_c})^2}}$$

To find the phase of TF:

$$\theta(\omega) = 0 - \tan^{-1}(\frac{\omega Rc}{1}) = \tan^{-1}(\frac{\omega}{\ }) $$

Example 3.3:

By using voltage divider:

$$V_o = V_i \frac{R}{R + \frac{1}{Sc}} \times SC$$

$$\frac{V_o}{V_i} = \frac{SRC}{1 + SRc} = \frac{j\omega Rc}{1 + j\omega Rc}$$

To find the pole of TF (Dnum = 0):

$$|T(S)| = \frac{1}{\sqrt{(1)^2 + (\omega Rc)^2}} = \frac{1}{\sqrt{2}}$$

$$\omega_c = \frac{1}{Rc}$$

To find the magnitude of TF:

$$|T(S)| = \frac{(\omega Rc)^2}{\sqrt{(1)^2 + (\omega Rc)^2}} \quad \text{use } \omega_c$$

$$|T(S)| = \frac{\frac{\omega}{\omega_c}}{\sqrt{1 + (\frac{\omega}{\omega_c})^2}}$$

To find the phase of TF:

$$\theta(\omega) = \frac{\pi}{2} - \tan^{-1}(\frac{\omega Rc}{1})$$

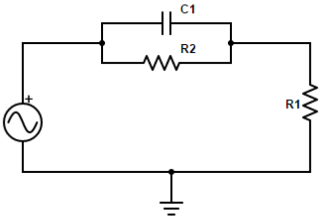
$$= \frac{\pi}{2} - \tan^{-1}(\frac{\omega}{\omega_c})$$

Example 3.4:

To realize $|T(0)| = 0.3$:

a pure capacitor will not meet this condition we

need to add shunt resistor, the exapectid cct.:



Back to Example34.

$$V_o = V_i \frac{ScR_1R_2 + R_2}{ScR_1R_2 + (R_1 + R_2)} \quad (4)$$

$$|T(S)|_{\omega=0} = \frac{R_1}{R_1 + R_2} \quad (5)$$

To find the pole of TF (Dnum = 0):

$$ScR_1R_2 + R_2 = 0 \Rightarrow \omega = \frac{1}{cR_1} \quad (6)$$

From 4 and 6 assume $c = 1\mu f$

$\therefore R_1 = 1K\Omega$ and $R_2 = 429\Omega$