Instructor : Dr. Mustafa M. Shiple
Subject: Shaping Circuits /(EEC 242)

Term: Feb / May 2015

ANSWER THE FOLLOWING QUESTIONS:

1. Design a Butterworth low-pass filter to realize the following specifications: [15 marks] $[C_o, A_m]$

$$\alpha_{max} = 0.5dB \text{ , } \alpha_{min} = 35dB \text{ , } \omega_p = 1000rad/s \text{ ,and } \omega_p = 3000rad/s.$$

(a) Calculate the filter order, pole locations in the S-plane,and transfer function.

(b) Calculate denormalized frequency, gain and quality factor for each pole.

(c) By keeping the gain of the third pole to unity, Calculate all filter components.

(d) Redesign LPF to use resistors with $8.1k\Omega$.

(e) Redesign 5^{th} order LPF (designed in part c)with $\omega_B = 12343.4rad/s$.

Solution:

(a)

$$n = \frac{\log\left(\frac{(10^{\frac{\alpha_{min}}{10}}-1)}{(10^{\frac{\alpha_{max}}{10}}-1)}\right)}{2\log\frac{\omega_S}{\omega_P}} = \frac{\log\left(\frac{(10^{\frac{35}{10}}-1)}{(10^{\frac{0.5}{10}}-1)}\right)}{2\log\frac{3000}{1000}} = 4.625 \approx 5$$

pole locations:$\psi = \frac{180}{n} = 36°$

- $0 \Rightarrow (S+1) \Rightarrow P_0 = -1$
- $\pm 36 \Rightarrow (S^2 + 2\cos 36 S + 1) = (S^2 + 0.618S + 1)$
  $\Rightarrow P_{1,2} = -\cos\psi + \pm j\sin\psi = -0.3 \pm J0.95$
- $\pm 72 \Rightarrow (S^2 + 2\cos 72 S + 1) = (S^2 + 1.618S + 1) \Rightarrow P_{3,4} = -0.8 \pm J0.58$

(b)

$$\epsilon = \sqrt{(10^{\frac{\alpha_{max}}{10}} - 1)} = 0.349$$

$$\omega_{BLPF} = \epsilon^{-\frac{1}{n}} \times \omega_P = 1234.34rad/s$$

Gain and quality factor: $S^2 + \frac{\omega_0}{Q}S + \omega_0^2$ and $k = 3 - \frac{1}{Q}$

Pole	**Term**	**Q**	**K**
0	$S+1$	0.5	1
1,2	$S^2 + 1.618S + 1$	0.618	1.382
3,4	$S^2 + 0.618S + 1$	1.618	2.382

[Total Marks is 40]

(c) Assume $C = 10nF$

$$R = \frac{1}{\omega_B C} = \frac{1}{1234.34 \times 10n} = 81k\Omega$$

$$K_{1,2} = 1 + \frac{R_A}{R_B} \Rightarrow R_B = \frac{R_A}{K_{1,2} - 1} = \frac{81K}{0.382} = 212k\Omega \qquad \text{Assume } R = R_A$$

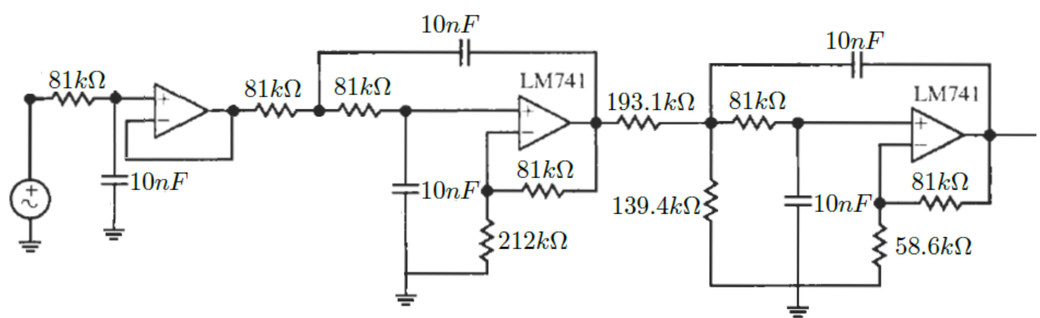
$$K_{3,4} = 1 + \frac{R_A}{R_B} \Rightarrow R_B = \frac{R_A}{K_{1,2} - 1} = \frac{81K}{1.382} = 58.6k\Omega \qquad \text{Assume } R = R_A$$

<u>Reduce</u> the third gain to be unity = 1

$$a_{3,4} = \frac{1}{2.382} = 0.419$$

$$R_1 = \frac{R}{a} = \frac{81k}{2.382} = 193.1k\Omega$$

$$R_2 = \frac{R}{1-a} = \frac{81k}{2.382} = 139.4k\Omega$$

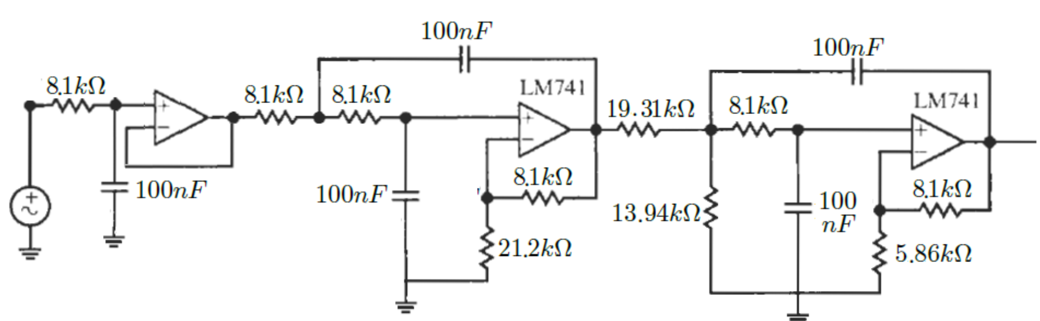


(d)

$$\Delta K_m = \frac{new_value}{old_value} = \frac{8.1k}{81k} = 0.1$$

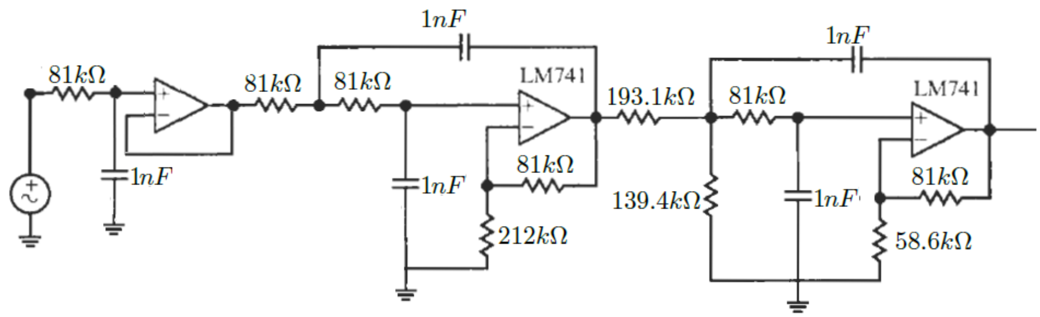
$$\forall R_{new} = 0.1 R_{old}$$

$$\forall C_{new} = \frac{C_{old}}{\Delta k_m} = \frac{10nF}{0.1} = 100nF$$



(e)

$$\Delta K_f = \frac{new_value}{old_value} = \frac{12343.4}{1234.34} = 10$$
$$\forall R_{new} = \text{not changed}$$
$$\forall C_{new} = \frac{C_{old}}{\Delta k_f} = \frac{10nF}{10} = 1nF$$



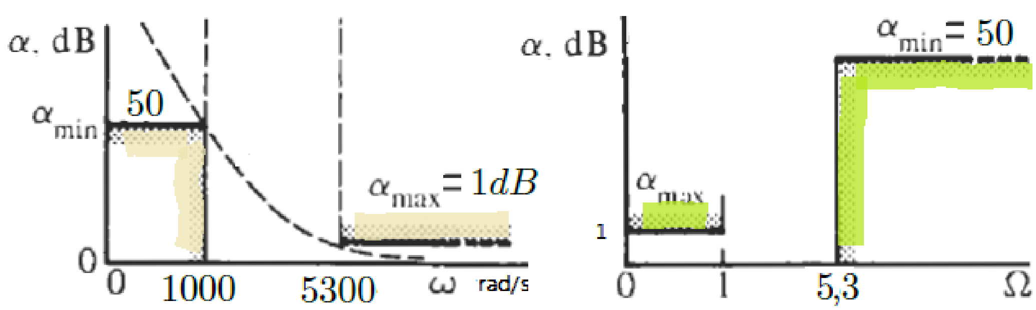
2. Design a Butterworth High-pass filter to realize the following specifications: [15 marks] [C_o, A_m]

$$\alpha_{max} = 1dB \text{ , } \alpha_{min} = 50dB \text{ , } \omega_p = 5300rad/s \text{ ,and } \omega_p = 1000rad/s.$$

(a) Calculate the filter order, pole locations in the S-plane,and transfer function.

(b) Calculate denormalized frequency, gain and quality factor for each pole.

Solution:
(a) transform HPF ⇒ LPF By replace $\forall\omega_{LPF} = \frac{\omega_{HPF_{pass}}}{\omega_{HPF}}$



$$n = \frac{\log\left(\frac{(10^{\frac{\alpha_{min}}{10}}-1)}{(10^{\frac{\alpha_{max}}{10}}-1)}\right)}{2\log\frac{\omega_S}{\omega_P}} = \frac{\log\left(\frac{(10^{\frac{50}{10}}-1)}{(10^{\frac{1}{10}}-1)}\right)}{2\log\frac{5.3}{1}} = 3.85 \approx 4$$

pole locations: $\psi = \frac{180}{n} = 45°$

- $\pm 22.5 \Rightarrow (S^2 + 2\cos 22.5S + 1) = (S^2 + 1.847S + 1)$
  $\Rightarrow P_{1,2} = -\cos\psi \pm j\sin\psi = -0.923 \pm J0.383$
- $\pm 67.5 \Rightarrow (S^2 + 2\cos 72S + 1) = (S^2 + .765S + 1) \Rightarrow P_{3,4} = -0.383 \pm J0.926$

(b)

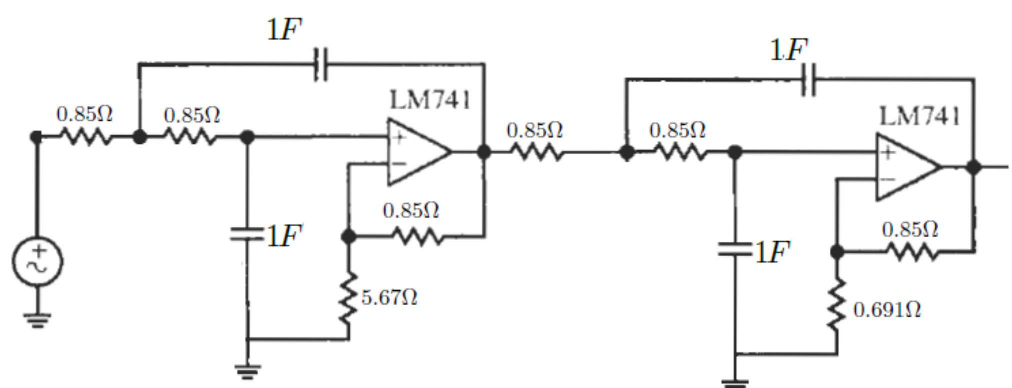
$$\epsilon = \sqrt{(10^{\frac{\alpha_{max}}{10}} - 1)} = 0.509$$
$$\omega_{BLPF} = \epsilon^{-\frac{1}{n}} \times \omega_P = 1.18 rad/s$$

Gain and quality factor: $S^2 + \frac{\omega_0}{Q}S + \omega_0^2$ and $k = 3 - \frac{1}{Q}$

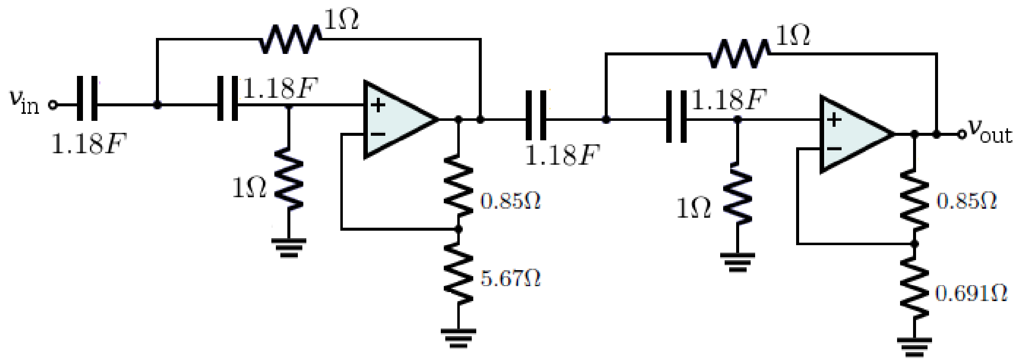
Pole	Term	Q	K
1,2	$S^2 + 1.847 + 1$	0.54	1.15
3,4	$S^2 + 0.765 + 1$	1.31	2.23

Assume $C = 1F$

$$R = \frac{1}{\omega_{BLPF}C} = \frac{1}{1.18 \times 1} = 0.85\Omega$$
$$K_{1,2} = 1 + \frac{R_A}{R_B} \Rightarrow R_B = \frac{R_A}{K_{1,2} - 1} = \frac{0.85}{0.15} = 5.67\Omega \qquad \text{Assume } R = R_A$$
$$K_{3,4} = 1 + \frac{R_A}{R_B} \Rightarrow R_B = \frac{R_A}{K_{1,2} - 1} = \frac{0.85}{1.23} = 0.691\Omega \qquad \text{Assume } R = R_A$$



Transform LPF $\Rightarrow$ HPF :
(1)RC to CR transformation:
$C_{new} = \frac{1}{R} = 1.18F \quad R_{new} = \frac{1}{C} = 1\Omega$



(2)De-normalization:
$\omega_{HLPF} = \frac{\omega_{HPF_{pass}}}{\omega_{LPF}} = \frac{5300}{1.18} = 4491.53 rad/s$
Assume $C = 10nF$

$$k_m = \frac{C_{old}}{c_{new} k_f} = \frac{1.18}{10n \times 4491.53} = 26.3K\Omega$$
$$R = 1 \times 26.3k = 26.3K\Omega$$
$$R_A = 0.85 \times 26.3k = 22.3K\Omega$$
$$R_{B1} = 5.67 \times 26.3k = 149K\Omega$$
$$R_{B2} = 6.91 \times 26.3k = 181.7K\Omega$$