# Bode Plot

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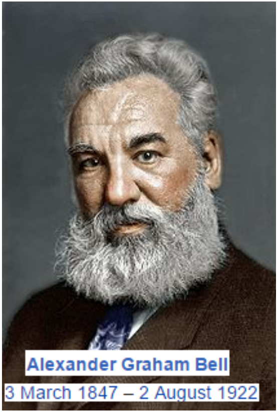
# Outline

# The Decibel

- Named for Alexander Graham Bell.
- Originally used to measure power losses in telephone lines.
- A Bel is the common log of the ratio of two power levels.
- A decibel is one-tenth of a bel.



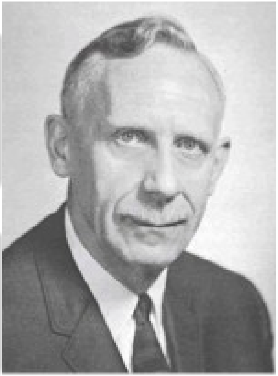
# The Bode Plot

### Definition

a graph of the frequency response of a system. It is usually a combination of a **Bode magnitude plot**, expressing the magnitude of the frequency response, and a **Bode phase plot**, expressing the phase shift.

### The General Form

$$|T(S)| = K\frac{(\frac{S}{z_0}+1)(\frac{S}{z_1}+1)\dots(\frac{S}{z_n}+1)}{(\frac{S}{p_0}+1)(\frac{S}{p_1}+1)\dots(\frac{S}{p_n}+1)}$$



Hendrik Wade Bode
1905 — 1982

# Expressing in dB

$$|T(S)| = K\frac{(\frac{S}{z_0}+1)(\frac{S}{z_1}+1)\dots(\frac{S}{z_n}+1)}{(\frac{S}{p_0}+1)(\frac{S}{p_1}+1)\dots(\frac{S}{p_n}+1)}$$

The magnitude:

$$20\log_{10}|T(S)| = 20\log K + 20\log(\frac{S}{z_0}+1) + 20\log(\frac{S}{z_n}+1) + \dots$$
$$-20\log(\frac{S}{p_n}+1) - 20\log(\frac{S}{p_n}+1)$$

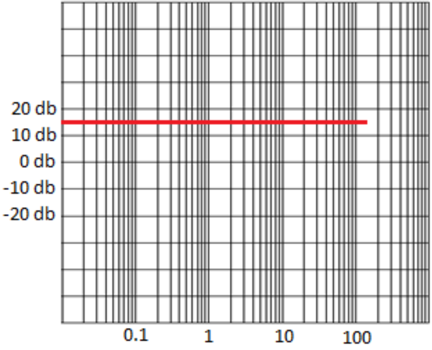
The phase:

$$\angle T(S) = \tan^{-1}(\frac{0}{K}) + \tan^{-1}(\frac{1}{z_0}) + \tan^{-1}(\frac{1}{z_n}) + \dots$$
$$-\tan^{-1}(\frac{1}{p_0}) - \tan^{-1}(\frac{1}{z_0})$$

# Bode plot of a constant

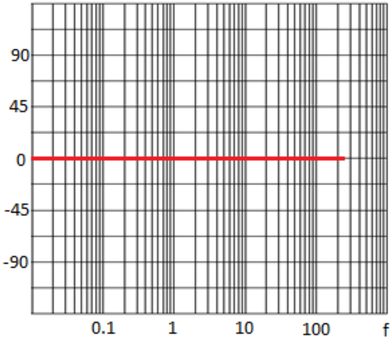
## Magnitude

$$20\log_{10}|T(S)| = 20\log K$$




## Phase

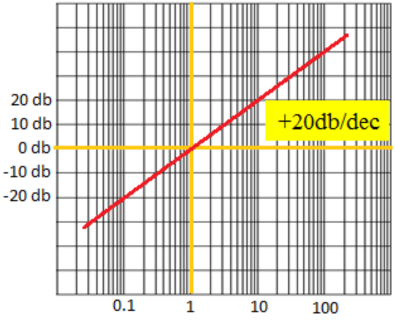
$$\angle T(S) = \tan^{-1}\left(\frac{0}{K}\right) = 0^{o}$$



# Bode plot of S as a zero

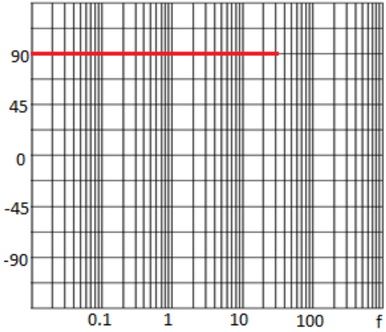
## Magnitude

$$|T(S)| = S$$
$$20\log_{10}|T(S)| = 20\log S$$




## Phase

$$\angle T(S) = \angle S = \tan^{-1}\left(\frac{\omega}{0}\right) = 90^o$$

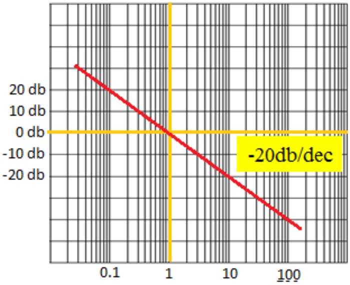


# Bode plot of S as a pole

## Magnitude

$$|T(S)| = (\frac{1}{S})$$

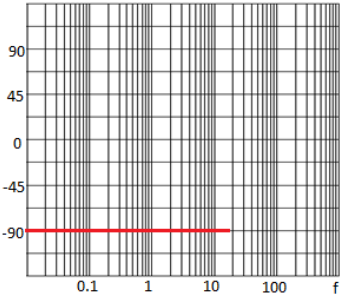
$$20\log_{10}|T(S)| = -20\log S$$




## Phase

$$\angle T(S) = \angle(\frac{1}{S}) = -\tan^{-1}\left(\frac{\omega}{0}\right)$$

$$= -90^o$$

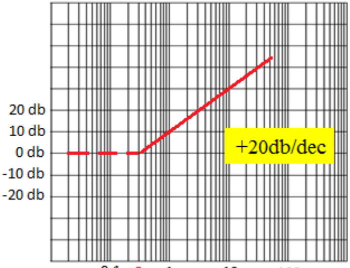


# Bode plot of of $(\frac{s}{z} + 1)$

## Magnitude

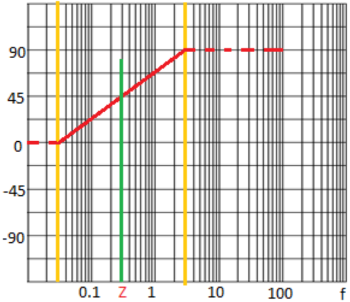
$$|T(S)| = \sqrt{\left(\frac{s}{z}\right)^2 + 1}$$

$$20\log_{10}|T(S)| = 20\log\left(\sqrt{\left(\frac{s}{z}\right)^2 + 1}\right)_{\omega=z}^{\omega=10z}$$




## Phase

$$\angle T(S) = \angle(\frac{s}{z} + 1) = \tan^{-1}\left(\frac{\omega}{z}\right)$$

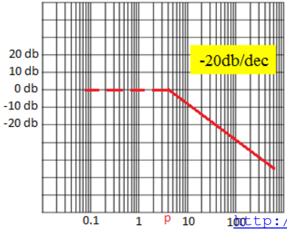


# Bode plot of $\frac{1}{(\frac{s}{p}+1)}$

## Magnitude

$$|T(S)| = \left(\frac{1}{\sqrt{(\frac{s}{p})^2+1}}\right)$$

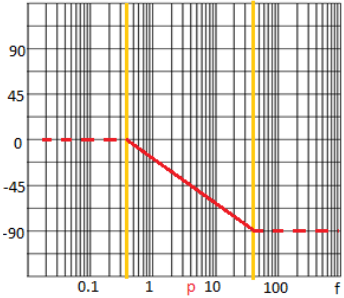
$$20\log|T(S)| = -20\log\left(\sqrt{\left(\frac{s}{p}\right)^2+1}\right)\Bigg|_{\omega=z}^{\omega=10z}$$




## Phase

$$\angle T(S) =$$

$$\angle(\frac{s}{p}+1)^{-1} = -\tan^{-1}\left(\frac{\omega}{p}\right)$$

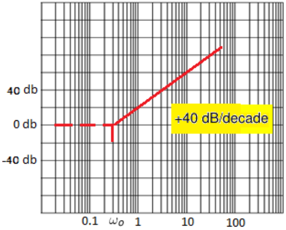


# Bode plot of $(\frac{s}{\omega_o})^2 + 2\zeta(\frac{s}{\omega_o}) + 1$

## Magnitude

+40 dB/decade

$$peak = 20log(2\zeta)$$

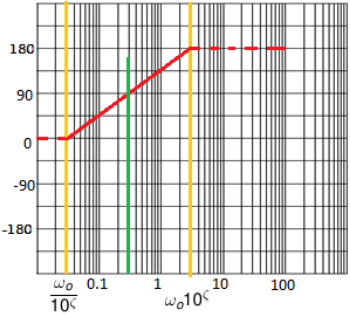



## Phase

0 to 180

$$Low\_frequency = \frac{\omega_o}{10^\zeta}$$

$$high\_frequency = \omega_o \times 10^\zeta$$

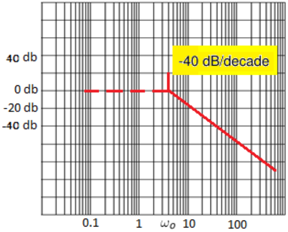


# Bode plot of $\frac{1}{(\frac{s}{\omega_o})^2 + 2\zeta(\frac{s}{\omega_o}) + 1}$

## Magnitude

-40 dB/decade

$$peak = 20log(2\zeta)$$

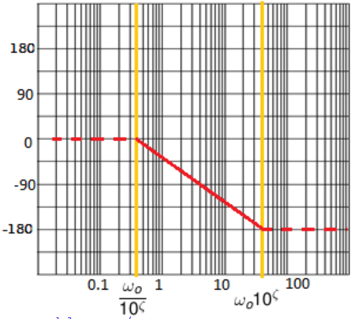



## Phase

0 to -180

$$Low\_frequency = \frac{\omega_o}{10^{\zeta}}$$

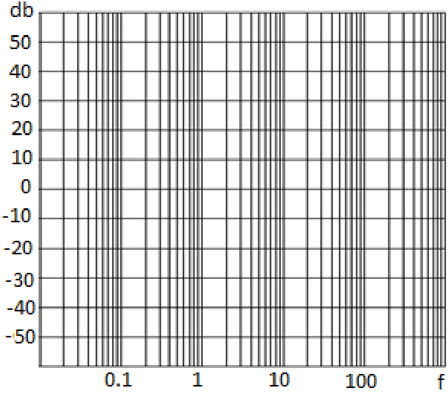
$$high\_frequency = \omega_o \times 10^{\zeta}$$

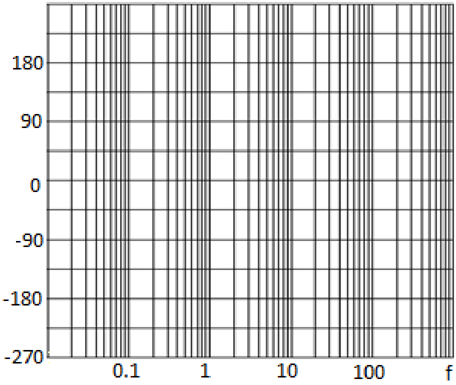


## Example 1:

Draw the bode plot of:

$$Y(j\omega) = \frac{5000(j\omega + 10)}{(j\omega + 1)(j\omega + 50)}$$



## Example 1:

$$Y(j\omega) = \frac{5000(j\omega + 10)}{(j\omega + 1)(j\omega + 50)}$$

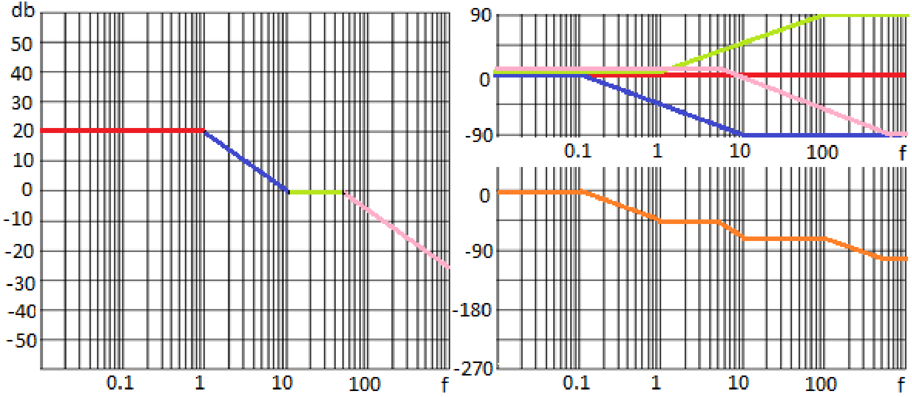
Ans:Rearrange the equation to the stander form

$$Y(j\omega) = 5000\frac{1}{10}\frac{1}{50}\frac{(j\dfrac{\omega}{10} + 1)}{(j\omega + 1)(j\frac{\omega}{50} + 1)} \qquad = 10\frac{(j\dfrac{\omega}{10} + 1)}{(j\omega + 1)(j\frac{\omega}{50} + 1)}$$

Order according frequencies:

- $K = 10 = 20db$ and $\angle k = 0^o$.
- $P_1 @ 1Hz$ drops off with a slope of -20 dB/dec. $\angle 0.1Hz = 0^o$ and drops linearly down to $-90^o @ 10Hz$.
- $Z_1 @ 10Hz$ rises with a slope of 20 dB/dec. $\angle 1Hz = 0^o$ and rises linearly up to $90^o @ 10Hz$.
- $P_2 @ 50Hz$ drops off with a slope of -20 dB/dec. $\angle 5Hz = 0^o$ and drops linearly down to $-90^o @ 500Hz$.

# Example 1:

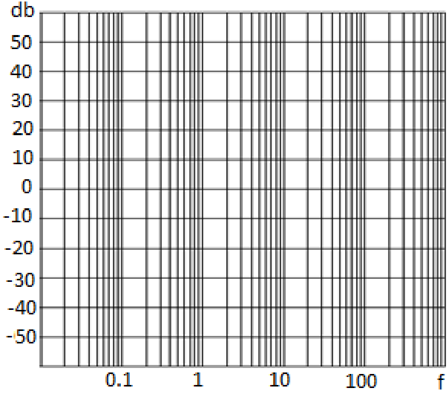


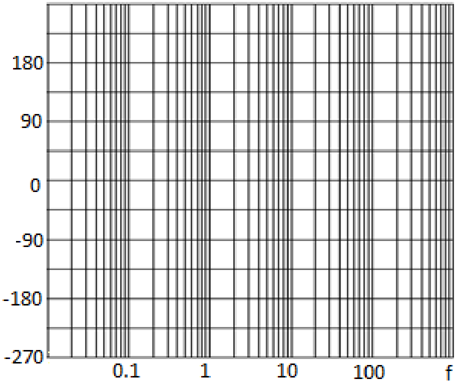

- $K = 10 = 20db$ and $\angle k = 0^o$.
- $P_1@1Hz$ drops off with a slope of -20 dB/dec. $\angle 0.1Hz = 0^o$ and drops linearly down to $-90^o@10Hz$.
- $Z_1@10Hz$ rises with a slope of 20 dB/dec. $\angle 1Hz = 0^o$ and rises linearly up to $90^o@10Hz$.
- $P_2@50Hz$ drops off with a slope of -20 dB/dec. $\angle 5Hz = 0^o$ and drops linearly down to $-90^o@500Hz$.

# Example 2:

Draw the bode plot of:

$$Y(s) = \frac{100(s+1)}{(s^2 + 110s + 1000)}$$

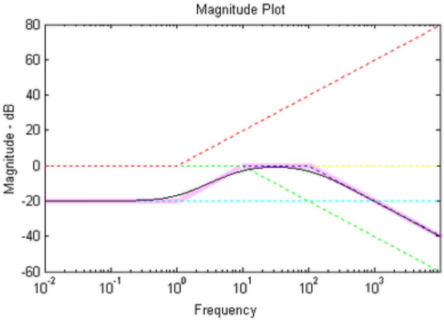


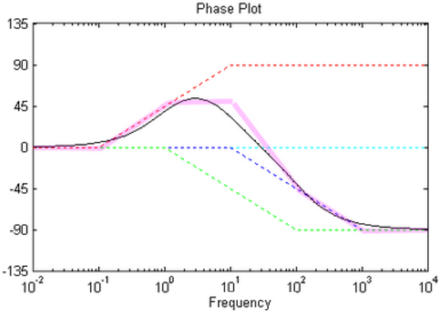



# Example 2:

Draw the bode plot of:

$$Y(s) = \frac{100(s+1)}{(s^2+110s+1000)} = \frac{100(s+1)}{(s+10)(s+100)} = 0.1\frac{(s+1)}{(\frac{s}{10}+1)(\frac{s}{100}+1)}$$

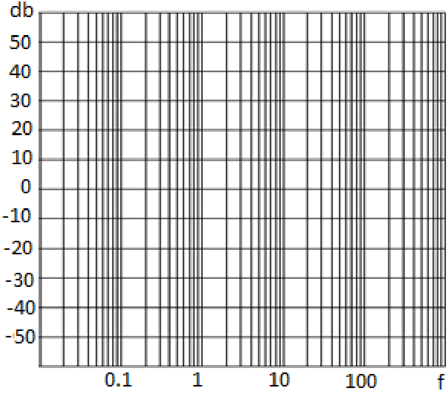


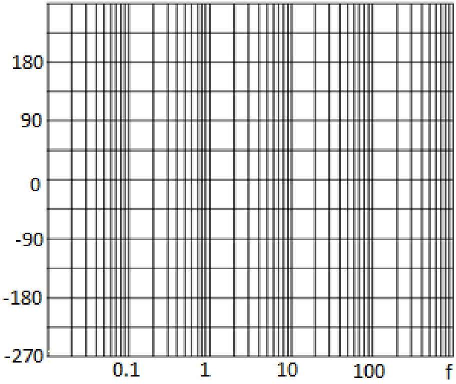



# Example 3:

Draw the bode plot of:

$$Y(s) = \frac{10(s+10)}{(s^2+3s)}$$



## Example 2:

Draw the bode plot of:

$$Y(s) = \frac{10(s+10)}{(s^2+3s)} = 33.3\frac{(\frac{s}{10}+1)}{s(\frac{s}{3}+1)}$$