Linear Wave Shaping (Low Pass Filter)

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Wave Shaping Circuits (EET 232), 2020

Outline

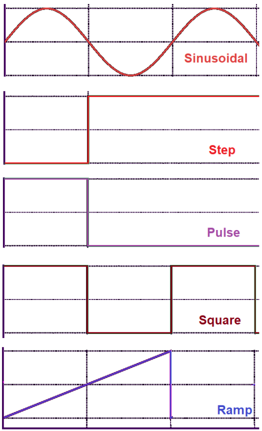
1. Introduction
2. Sinusoidal wave
3. Step Signal
4. Ramp Signal
5. Exponential Input
6. Integrator

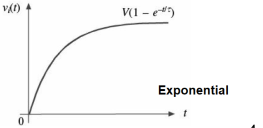
Section 1

Introduction

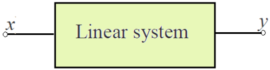
Linear Wave Shaping

The process whereby the form of a non-sinusoidal signal is altered by transmission through a linear network is called linear wave shaping.





Linear network (System):Properties



1. **Homogeneity:** (Multiply by constant) for the input nx to the system, the corresponding output is ny (where n is an integer).
2. **Additivity:** This property is called additivity. Homogeneity and additivity, taken together, comprise the principle of superposition. [ex: $(y_1 + y_2) = x_1 + x_2$].
3. **Shift invariance:** [ex: $(y(t-a)) = x(t-a)$]

Section 2

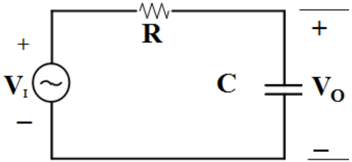
Sinusoidal wave

Low Pass filter

By using voltage divider:

$$V_o = V_i \frac{\frac{1}{Sc}}{R + \frac{1}{Sc}} \qquad \times SC$$

$$\frac{V_o}{V_i} = \frac{1}{1 + SRc} = \frac{1}{1 + j\omega Rc}$$



To find cut off frequency, the input signal loses half power or $\frac{1}{\sqrt{2}}$ voltages

$$|T(S)| = \frac{1}{\sqrt{1 + (\frac{\omega}{\omega_c})^2}}$$

$$|T(S)| = \frac{1}{\sqrt{(1)^2 + (\omega Rc)^2}} = \frac{1}{\sqrt{2}}$$

$$\omega_c = \frac{1}{Rc}$$

To find the phase of TF:

$$\theta(\omega) = \tan^{-1}(\frac{0}{1}) - \tan^{-1}(\frac{\omega Rc}{1})$$

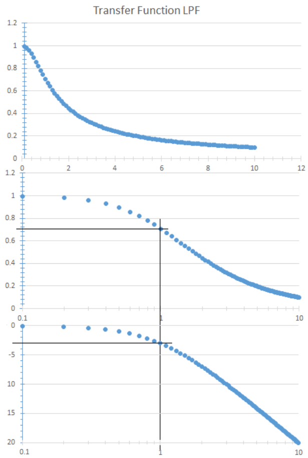
$$= 0 - \tan^{-1}(\frac{\omega}{\omega_c}) = -\tan^{-1}(\frac{\omega}{\omega_c})$$

$|T(S)|$

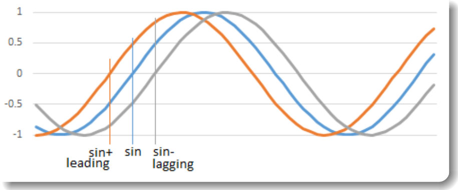
$$\alpha = 20 log_{10} |T(S)|$$

$$f_c = \frac{1}{2\pi Rc} = \frac{1}{2 \times 3.14 \times 12k \times 100} = 132.6 Hz$$

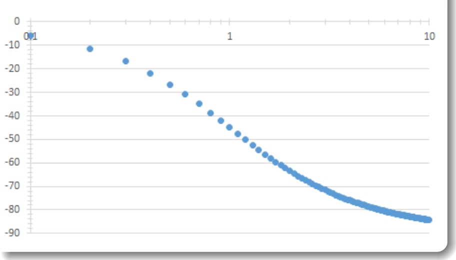
Magnitude



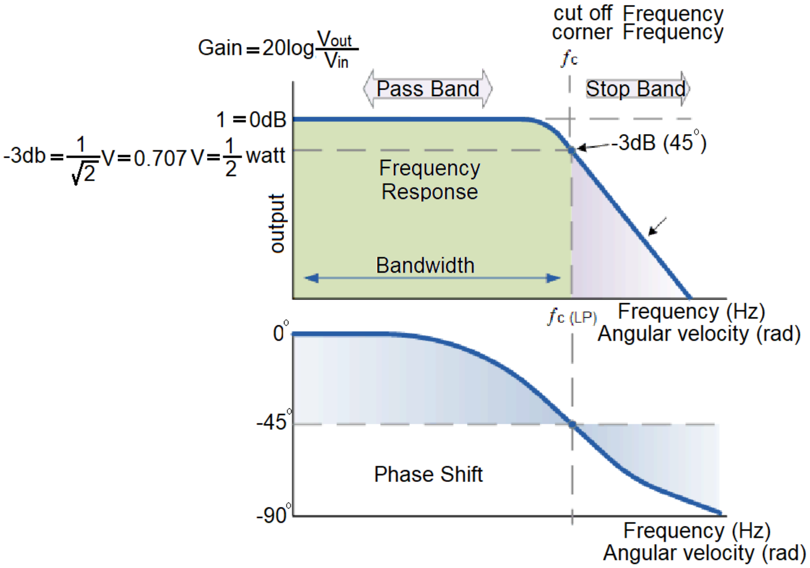
$< \theta(\omega)$



Phase



Bode Plot

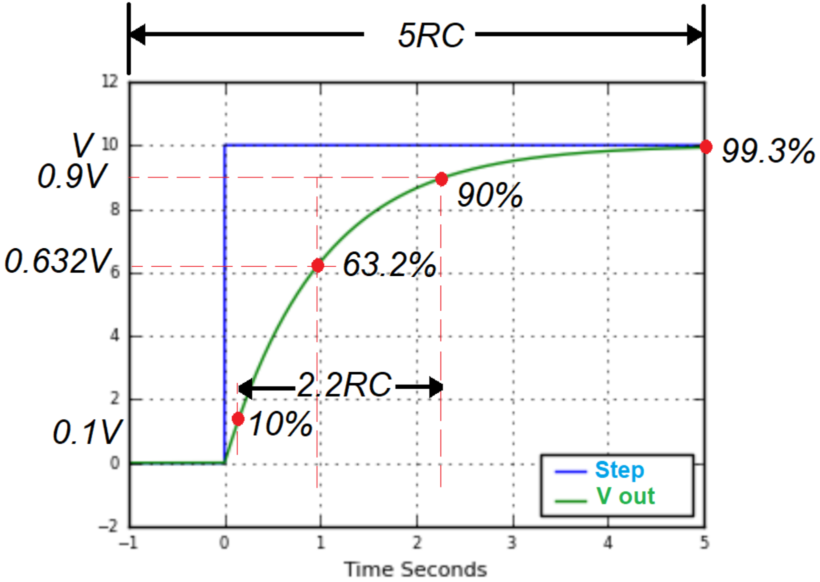


cut off Frequency
corner Frequency
$Gain = 20\log\frac{V_{out}}{V_{in}}$
f_c
Pass Band
Stop Band
1 = 0dB
$-3db = \frac{1}{\sqrt{2}}V = 0.707\,V = \frac{1}{2}\,watt$
-3dB (45°)
Frequency Response
output
Bandwidth
f_c (LP)
Frequency (Hz)
Angular velocity (rad)
0°
-45°
Phase Shift
-90°
Frequency (Hz)
Angular velocity (rad)

Section 3

Step Signal

Step Voltage Input



Section 4

Ramp Signal

Back to basic concepts

$$v_i = \alpha t = \frac{\alpha}{S^2}$$

$$V_o = v_i \frac{1}{1+SRC} = \frac{\alpha}{S^2}\frac{1}{(1+SRC)}$$

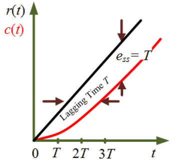
$$= \frac{?}{S} + \frac{?\alpha}{S^2} + \frac{?}{(1+SRC)}$$

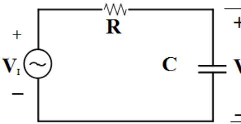
$$= \frac{-\alpha RC}{S} + \frac{\alpha}{S^2} + \frac{\alpha (RC)^2}{(1+SRC)}$$

last argument should be formed as table

$$= \left[\frac{-\alpha RC}{S} + \frac{\alpha}{S^2} + \frac{\alpha (RC)}{(\frac{1}{RC}+S)} \right]$$

$$= -\alpha RC + \alpha t + \alpha (RC) e^{-\frac{t}{RC}}$$





	$f(t)$	$F(s)$
1.	Unit impulse $\delta(t)$	1
2.	Unit step $1(t)$	$\frac{1}{s}$
3.	t	$\frac{1}{s^2}$
4.	e^{-at}	$\frac{1}{s+a}$
5.	te^{-at}	$\frac{1}{(s+a)^2}$

Back to basic concepts

$$V_o = -\alpha RC + \alpha t + \alpha(RC)e^{-\frac{t}{RC}}$$

RC very small

$$\begin{aligned} &= -\alpha RC + \alpha t + \alpha(RC)e^{-\frac{t}{RC}} \\ &= -\alpha RC + \alpha t = \alpha(t - RC) \end{aligned}$$

RC not small

Recall : $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \ldots$

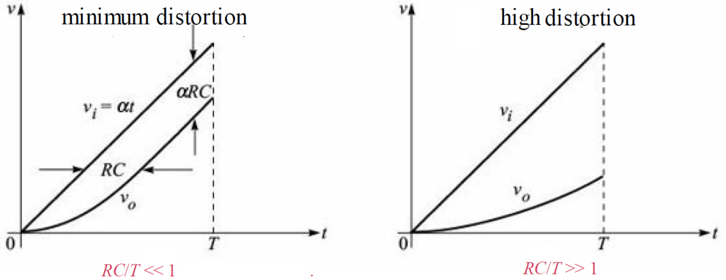
$$\begin{aligned} &= -\alpha RC + \alpha t + \alpha(RC)e^{-\frac{t}{RC}} \\ &= -\alpha RC + \alpha t + \alpha(RC)[1 - \frac{t}{RC} + \frac{t^2}{(RC)^2}] \\ &= -\alpha RC + \alpha t + \alpha(RC) - t\alpha + \alpha\frac{t^2}{2(RC)} \\ &= \frac{\alpha}{(RC)}\frac{t^2}{2} \end{aligned}$$

Error Ratio

$$= \frac{\alpha}{(RC)} \frac{t^2}{2}$$

error between input and output at small RC values:

$$e_t = \frac{\alpha t - (-\alpha RC + \alpha t)}{\alpha t} = \frac{RC}{t} = \frac{1}{\omega_c T} = \frac{1}{2\pi f_c T}$$



Example

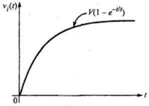
Consider 2 ms ramp signal input the output signal should has less than 0.1% error. Calculate the maximum yield f.
Solution:

$$e_t = \frac{1}{2\pi f_c T} \Rightarrow f_c = \frac{1}{2\pi e_t T} > 79.6KHz$$
$$\omega_c = \frac{1}{e_t T} = \frac{1}{RC} \Rightarrow RC < 2\mu S$$

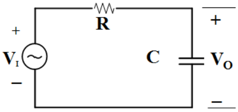
Section 5

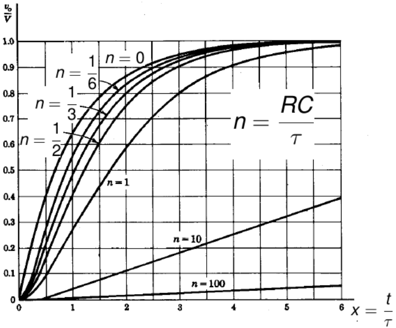
Exponential Input

Exponential Response



Exponential input.

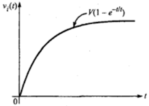




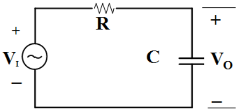
$$v_{in}(t) = V(1 - e^{\frac{-t}{\tau}})$$

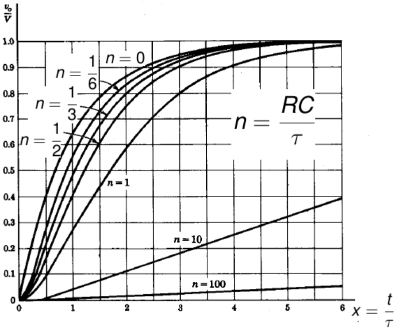
$$V_{out}(t) = V\left[1 - \frac{e^{(\frac{-t}{\tau})}}{1-n} + \frac{e^{(\frac{-t}{RC})}}{\frac{1}{n}-1}\right]$$

Exponential Response



Exponential input.





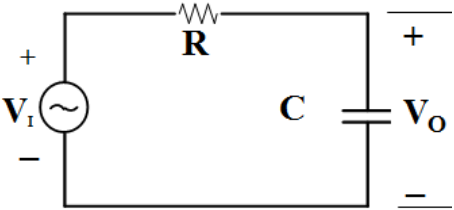
$$V_{out}(t) = \begin{cases} V\left[1 - \dfrac{e^{(-x)}}{1-n} + \dfrac{ne^{\left(\frac{-x}{n}\right)}}{1-n}\right] & n \neq 1 \\ 1-(1+x)e^{-x} & n = 1 \end{cases}$$

$$t_r = 1.05\sqrt{(2.2RC)^2 + (2.2\tau)^2}$$

Section 6

Integrator

Concepts



at charging if $RC >> T$, where: T is the period of the input signal.

$$\because RC >> T \text{ the capacitor never charge}$$

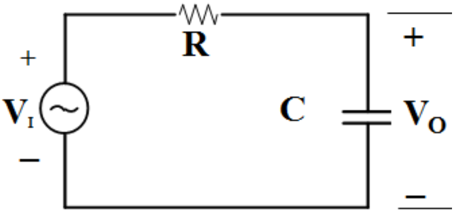
$$\therefore V_R >> V_C$$

$$V_{in} = V_R + V_C \Rightarrow V_{in} \approx V_R$$

$$V_o = V_C = \frac{1}{C}\int i(t)dt \approx \frac{1}{C}\int \frac{V_R}{R}dt$$

$$= \frac{1}{RC}\int V_{in}dt$$

Condition



- at charging if $RC >> T$, where: T is the period of the input signal.
- RC > 15T sine wave will be shifted $89^\circ \approx 90^\circ$

Difference betwee LPF VS Miller

Frequency	Low pass filter: Hs=1/(s+1)	Pure integrator: Hs=1/s
for f=1000hz	1/(6284+1)=1.5910899e-4	1/6284=1.5913431e-4
1Hz	1/(1+1)=1/2=0.500	1/(1)=1
DC	1/(1+0)=1	∞
step input	$f(t) = 1 - e^{(-t)}$	t