Linear Wave Shaping (High Pass Filter)

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Wave Shaping Circuits (EET 232), 2020

Outline

1. **Sinusoidal wave**
2. **Step Signal**
3. **Pulse Signal**
4. **Square Signal**
5. **Ramp Signal**
6. **Exponential Input**
7. **Differentiator**

Section 1

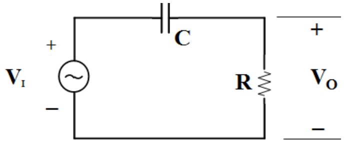
Sinusoidal wave

High Pass filter

By using voltage divider:

$$V_o = V_i \frac{R}{R + \frac{1}{Sc}} \times SC$$

$$\frac{V_o}{V_i} = \frac{SRc}{1 + SRc} = \frac{j\omega Rc}{1 + j\omega Rc}$$



To find cut off frequency, the input signal loses half power or $\frac{1}{\sqrt{2}}$ voltages

$$|T(S)| = \frac{\frac{\omega}{\omega_c}}{\sqrt{1 + (\frac{\omega}{\omega_c})^2}}$$

$$|T(S)| = \frac{\omega Rc}{\sqrt{(1)^2 + (\omega Rc)^2}} = \frac{1}{\sqrt{2}}$$

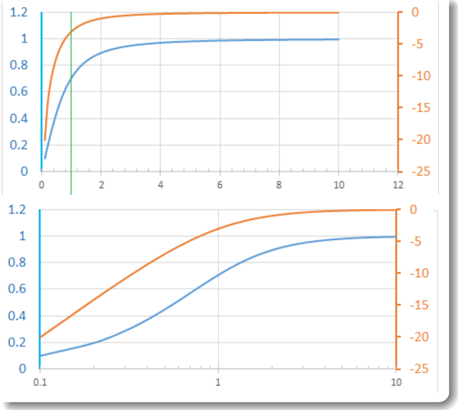
$$\omega_c = \frac{1}{Rc}$$

To find the phase of TF:

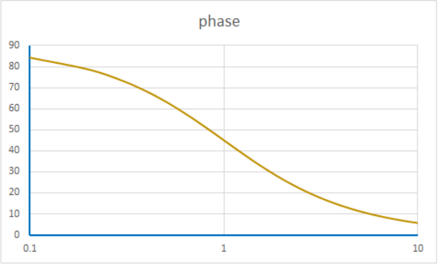
$$\theta(\omega) = \tan^{-1}(\frac{\omega Rc}{0}) - \tan^{-1}(\frac{\omega Rc}{1})$$

$$= 90 - \tan^{-1}(\frac{\omega}{\omega_c})$$

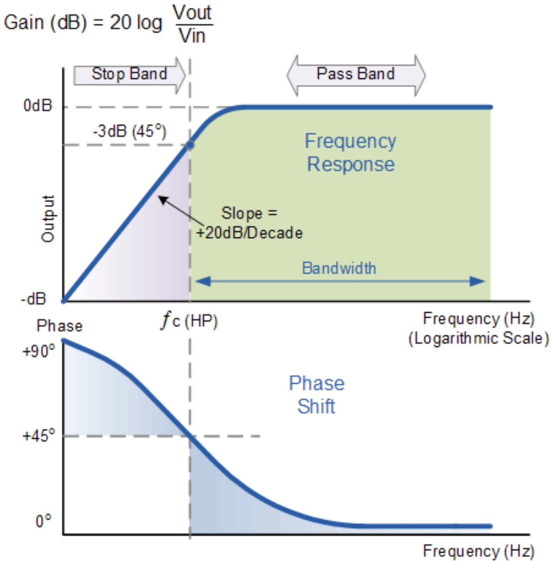
$< \theta(\omega)$



Phase



Bode Plot


Gain (dB) = 20 log Vout/Vin
Stop Band
Pass Band
0dB
-3dB (45°)
Frequency Response
Output
Slope = +20dB/Decade
Bandwidth
-dB
Phase
ƒc (HP)
Frequency (Hz) (Logarithmic Scale)
+90°
Phase Shift
+45°
0°
Frequency (Hz)

Section 2

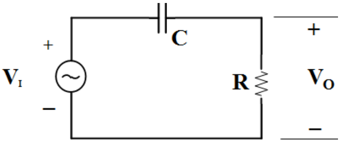
Step Signal

Back to basic concepts

$$v_i = \frac{1}{C}\int idt + v_o$$
$$= \frac{1}{C}\int \frac{v_o}{R}dt + v_o = \frac{1}{RC}\int v_o dt + v_o$$

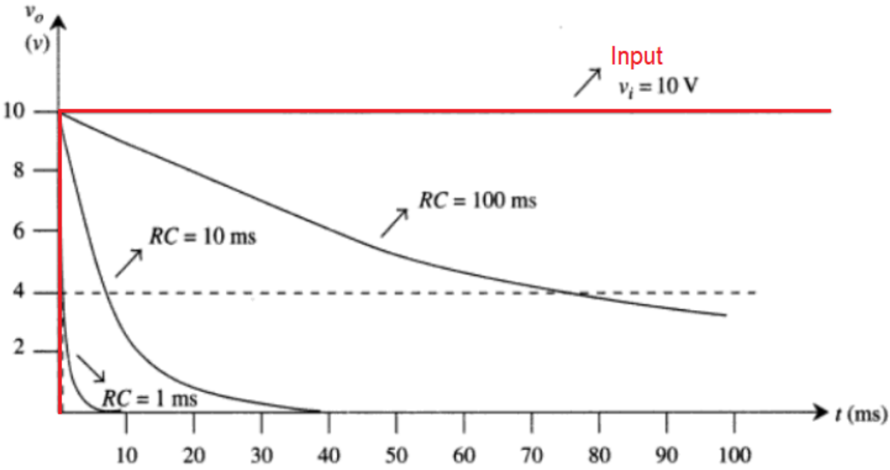
Laplace transform : (input = unit step)

$$\frac{V}{s} = \frac{1}{RC}\frac{v_o(s)}{s} + v_o(s)$$
$$v_o(s) = V\left(\frac{1}{s(\frac{1}{RCs}+1)}\right) = V\left(\frac{1}{\frac{1}{RC}+s}\right)$$
$$v_o(t) = Ve^{-\frac{t}{\tau}}$$



	$f(t)$	$F(s)$
1.	Unit impulse $\delta(t)$	1
2.	Unit step $1(t)$	$\frac{1}{s}$
3.	t	$\frac{1}{s^2}$
4.	e^{-at}	$\frac{1}{s+a}$
5.	te^{-at}	$\frac{1}{(s+a)^2}$

Step response

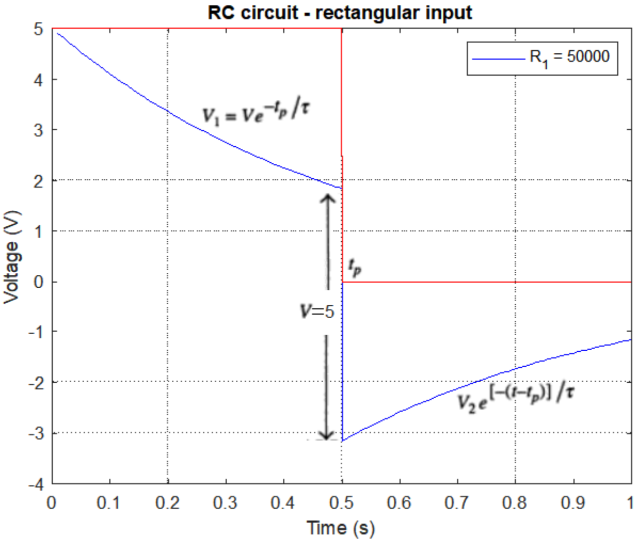


Step response fro different $\tau = RC$

Section 3

Pulse Signal

Pulse response

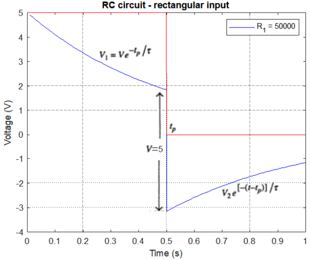


Pulse response features

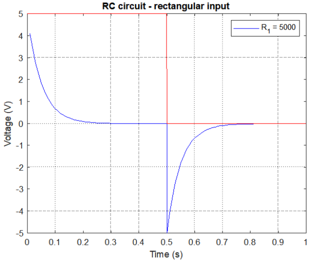
- Distortion ratio α^{-1} time constant τ.
- The output comprises a positive spike at beginning of th input pulse and negative at the end.
- $v_o =$

$$\begin{cases} Ve^{-\frac{t}{\tau}} & 0 < t < t_p \\ V\left(e^{-\frac{t_p}{\tau}} - 1\right) e^{-\frac{t - t_p}{\tau}} & t > t_p \end{cases}$$

low distortion



high distortion



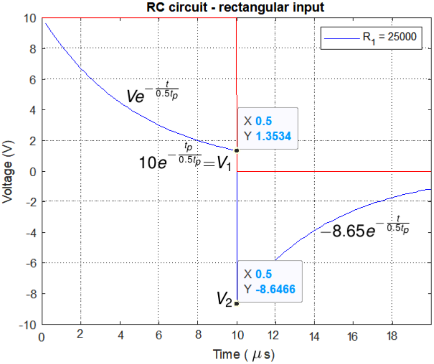
Pulse response:Example 1

A Pulse of 10V and duration $10\mu S$ is applied to HPF. Sketch the output for $RC = 0.5t_p$.

$$v_o(t) = Ve^{-\frac{t}{0.5tp}} = 10e^{-2} = 1.35V = V_1$$

$$V_2 = V_1 - V = 1.35 - 10 = -8.65V$$

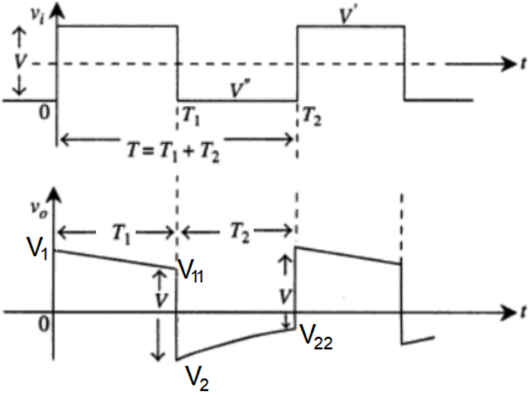
$$v_o = -8.65e^{-\frac{t}{0.5tp}}$$



Section 4

Square Signal

Square Response



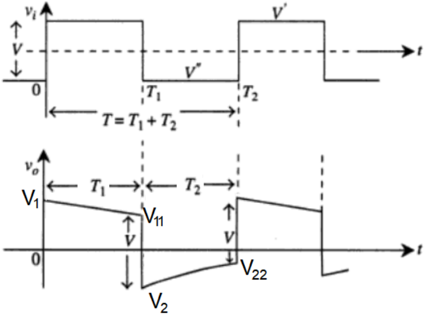
1. Symmetric square, $T_1 = T_2$.
2. As the capacitor blocks the DC, the DC component in the output is zero.

Square Response

For any square wave:

$$V_{11} = V_1 e^{-\frac{T_1}{\tau}}$$

$$V_2 = V_{11} - V \qquad V_{22} = V_2 e^{-\frac{T_2}{\tau}}$$



For symmetric square wave:

- $T_1 = T_2$.
- $V_1 = -V_2$
- $V_{11} = -V_{22}$

$$V_2 = V_{11} - V \Rightarrow V_2 = V_1 e^{-\frac{T_1}{\tau}} - V$$

$$\because V_1 = -V_2 \Rightarrow V_1 = -V_1 e^{-\frac{T_1}{\tau}} + V$$

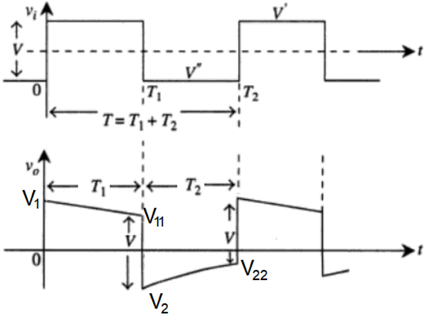
$$V_1 = \frac{V}{(1 + e^{-\frac{T_1}{\tau}})} = \frac{V}{(1 + e^{-\frac{T}{2\tau}})}$$

Square Response

$$V_{11} = V_1 e^{-\frac{T_1}{\tau}}$$

$$V_2 = V_{11} - V \qquad V_{22} = V_2 e^{-\frac{T_2}{\tau}}$$

$$\boxed{V_1 = \frac{V}{(1 + e^{-\frac{T}{2\tau}})}}$$



Square Response: Example 1

A 10 Hz square wave whose peak-to-peak amplitude is 2 V is fed to an amplifier. Calculate and plot the output waveform if the lower 3-dB frequency is 0.3 Hz.

Solution:

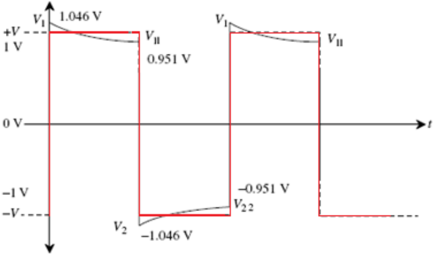
$$T = \frac{1}{f} = \frac{1}{10} = 0.1s$$

$$\omega_o = \frac{1}{RC} \Rightarrow RC = \frac{1}{\omega_o} = \frac{1}{2\pi 0.3}$$

$$\tau = RC = 0.53s$$

$$V_1 = \frac{V}{(1 + e^{-\frac{T}{2\tau}})} = 1.046V$$

$$V_{11} = V_1 e^{-\frac{T}{2\tau}} = 1.046e^{-\frac{0.1}{2\times 0.53}} = 0.951V$$



$$V_2 = -V_1 = -1.046V$$

$$V_{22} = -V_{11} - 0.951V$$

Square Response: Example 2

Given :$T_1 = 0.2s$, $T_2 = \tau = 0.4s$
calculate minimum and maximum values of the output waveform?
Solution:

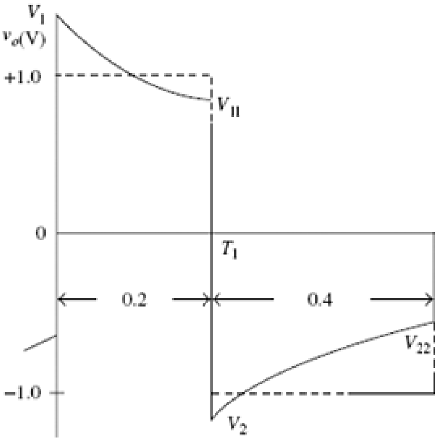
$$V_{11} = V_1 e^{-\frac{T}{\tau}} = V_1 e^{-\frac{0.2}{0.4}} = 0.606\,V_1$$

$$V_{22} = V_2 e^{-\frac{T}{\tau}} = V_2 e^{-\frac{0.4}{0.4}} = 0.367\,V_2$$

$$\because V_{11} - V_2 = 2 \quad , V_1 - v_{22} = 2$$

$$0.606\,V_1 - V_2 = 2 \qquad (1)$$

$$V_1 - 0.367\,V_2 = 2 \qquad (2)$$



By solving (1) and (2)
$V_1 = 1.628\,V$, $V_2 = -1.016\,V$
and
$V_{11} = 0.986\,V$, $V_{22} = -0.372\,V$

Section 5

Ramp Signal

Back to basic concepts

$$v_i = \alpha t = \frac{\alpha}{S^2}$$

$$V_o = v_i \frac{SRC}{1+SRC} = \frac{\alpha}{S}\frac{RC}{(1+SRC)}$$

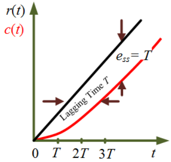
$$= \frac{?\alpha RC}{S} + \frac{?}{(1+SRC)}$$

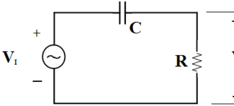
$$= \frac{\alpha RC}{S} + \frac{-\alpha (RC)^2}{(1+SRC)}$$

last argument should be formed as table

$$= \left[\frac{\alpha RC}{S} - \frac{\alpha (RC)}{(\frac{1}{RC}+S)}\right]$$

$$= \alpha RC - \alpha (RC) e^{-\frac{t}{RC}}$$





	$f(t)$	$F(s)$
1.	Unit impulse $\delta(t)$	1
2.	Unit step $1(t)$	$\frac{1}{s}$
3.	t	$\frac{1}{s^2}$
4.	e^{-at}	$\frac{1}{s+a}$
5.	te^{-at}	$\frac{1}{(s+a)^2}$

Back to basic concepts

$$V_o = \alpha RC - \alpha(RC)e^{-\frac{t}{RC}} = \alpha RC(1 - e^{-\frac{t}{RC}})$$

$RC << T$:

$$= \alpha RC - \alpha(RC)e^{-\frac{t}{RC}}$$
$$= \alpha RC$$

$(T/RC) \approx 0$

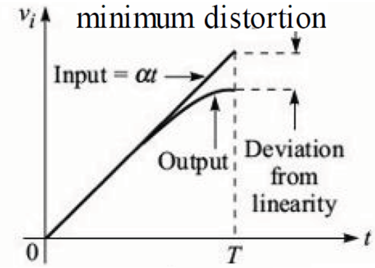
Recall : $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \ldots$

$$= \alpha RC - \alpha(RC)e^{-\frac{t}{RC}}$$
$$= \alpha RC - \alpha(RC)[1 - \frac{t}{RC} + \frac{t^2}{(RC)^2}]$$
$$= \alpha RC - \alpha(RC) + t\alpha - \alpha\frac{t^2}{2(RC)}$$
$$= \alpha t(1 - \frac{t}{2(RC)})$$

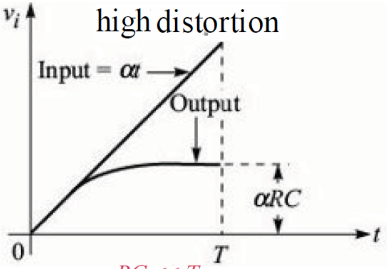
Transmission error

$$e_t = \frac{v_i - v_o}{v_i} = \left.\frac{\alpha t - \alpha t \left(1 - \frac{t}{2RC}\right)}{\alpha t}\right|_{t=T} = \pi f_c T$$

where



RC >> T



RC << T.

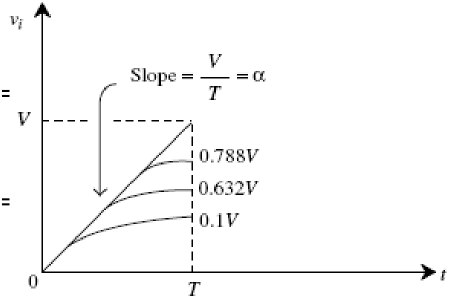
Ramp Response Summary

$$V_o(t) = \begin{cases} & \alpha RC(1 - e^{-\frac{t}{RC}}) \\ RC << T & \alpha RC \\ (T/RC) \approx 0 & \alpha t(1 - \frac{t}{2(RC)}) \end{cases}$$

Ramp Response: Example 1

A ramp is applied to an RC differentiator, as shown . Draw to scale the output waveform for the following cases: (i) T = RC, (ii) T = 0.5RC, (iii) T = 10RC.

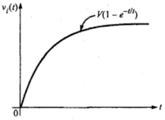
(i)$RC = T \qquad \alpha RC(1 - e^{-\frac{t}{RC}})$:
$V_o(t) = \frac{V}{T} RC(1 - e^{-\frac{t}{RC}}) =$
$V(1 - e^{-1}) = 0.632V$
================================
(ii)$RC << T \qquad \alpha RC$
$V_o(t) = 2V$
================================
(iii)$RC >> T \qquad \alpha RC(1 - e^{-\frac{t}{RC}})$
$V(0.1)(1 - e^{-10})$
@ t=T $\Rightarrow v(t) = 0.1V$



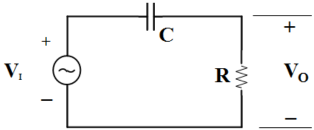
Section 6

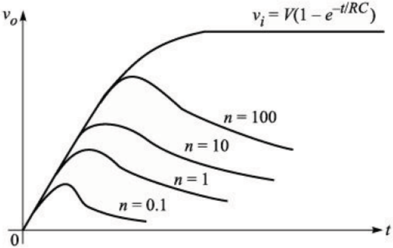
Exponential Input

Exponential Response



Exponential input.





$$v_{in}(t) = \frac{q}{C} + V_o$$

$$i = \frac{dq}{dt} \qquad V_R = iR$$

$$\frac{dv_{in}(t)}{dt} = \frac{dq}{Cdt} + \frac{dV_o}{dt} = \frac{i}{C} + \frac{dV_o}{dt}$$

$$\frac{dv_{in}(t)}{dt} = \frac{V_o}{\tau} + \frac{dV_o}{dt}$$

Exponential Response

$$\because v_{in} = V(1 - e^{\frac{-t}{\tau_e}}) \quad \therefore \frac{dv_{in}(t)}{dt} = \frac{V}{\tau_e} e^{\frac{-t}{\tau_e}}$$

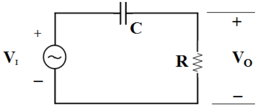
$$\frac{dv_{in}(t)}{dt} = \frac{V_o}{\tau} + \frac{dV_o}{dt}$$

$$\frac{V}{\tau_e} e^{\frac{-t}{\tau_e}} = \frac{V_o}{\tau} + \frac{dV_o}{dt}$$

$$\frac{V}{\tau_e} \frac{1}{S + \frac{1}{\tau_e}} = \frac{V_o(S)}{\tau} + SV_o(S)$$

$$\frac{V}{\tau_e} \frac{1}{S + \frac{1}{\tau_e}} = V_o(S) \left(\frac{1}{\tau} + S \right)$$

$$V_o(S) = \frac{V/\tau_e}{\left(S + \frac{1}{\tau_e}\right)\left(S + \frac{1}{\tau}\right)}$$



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1.	Unit impulse $\delta(t)$	1
2.	Unit step $1(t)$	$\frac{1}{s}$
3.	t	$\frac{1}{s^2}$
4.	e^{-at}	$\frac{1}{s+a}$
5.	te^{-at}	$\frac{1}{(s+a)^2}$

Exponential Response

$$V_o(S) = \frac{V/\tau_e}{\left(S + \frac{1}{\tau_e}\right)\left(S + \frac{1}{\tau}\right)}$$

$$\frac{A}{\left(S + \frac{1}{\tau_e}\right)} + \frac{B}{\left(S + \frac{1}{\tau}\right)} = \frac{V/\tau_e}{\left(S + \frac{1}{\tau_e}\right)\left(S + \frac{1}{\tau}\right)}$$

$$V/\tau_e = A\left(S + \frac{1}{\tau}\right) + B\left(S + \frac{1}{\tau_e}\right)$$

$$\text{Cancel parameter B} \Rightarrow S = -\frac{1}{\tau_e}$$

$$\therefore A = \frac{V/\tau_e}{\left(\frac{1}{\tau_e} - \frac{1}{\tau}\right)} = \frac{V}{\left(\frac{\tau_e}{\tau} - 1\right)}$$

Exponential Response

In the same way Cancel parameter A $\Rightarrow S = -\frac{1}{\tau}$

$$\therefore B = \frac{V/\tau_e}{\left(\frac{1}{\tau} - \frac{1}{\tau_e}\right)} = \frac{-V}{\left(\frac{\tau_e}{\tau} - 1\right)}$$

$$V_o(S) = \frac{V}{\left(\frac{\tau_e}{\tau} - 1\right)\left(S + \frac{1}{\tau_e}\right)} + \frac{-V}{\left(\frac{\tau_e}{\tau} - 1\right)\left(S + \frac{1}{\tau}\right)}$$

$$V_o(t) = \frac{V}{\left(\frac{\tau_e}{\tau} - 1\right)}\left[e^{\frac{-t}{\tau_e}} - e^{\frac{-t}{\tau}}\right]$$

$$= \begin{cases} \tau >> \tau_e, & \frac{-V}{\left(\frac{\tau_e}{\tau} - 1\right)}\left[e^{\frac{-t}{\tau}}\right] \\ \tau = \tau_e, & \frac{V/\tau_e}{\left(S + \frac{1}{\tau_e}\right)^2} = \frac{V}{\tau_e} t e^{\frac{-t}{\tau_e}} \end{cases}$$

Exponential Response

$$V_o(t) = \begin{cases} \tau \neq \tau_e, & \frac{V}{\left(\frac{\tau_e}{\tau} - 1\right)} \left[e^{\frac{-t}{\tau_e}} - e^{\frac{-t}{\tau}} \right] \\ \tau >> \tau_e, & \frac{-V}{\left(\frac{\tau_e}{\tau} - 1\right)} \left[e^{\frac{-t}{\tau}} \right] \\ \tau = \tau_e, & \frac{V/\tau_e}{\left(S + \frac{1}{\tau_e}\right)^2} = \frac{V}{\tau_e} t e^{\frac{-t}{\tau_e}} \end{cases}$$

Exponential Response: Example 1

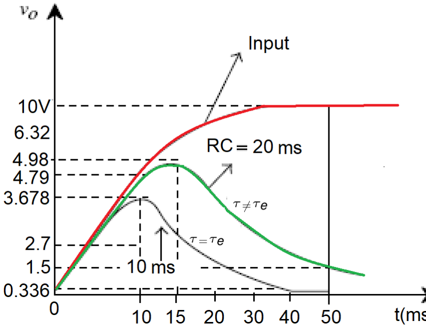
An exponential input of $10(1 - e^{-\frac{t}{10\times10^{-3}}})$ is applied to a high-pass RC circuit. Plot the response when (i) RC = 20 ms and (ii) RC = 10 ms.

$\tau \neq \tau_e$:

$$\frac{V}{\left(\frac{\tau_e}{\tau} - 1\right)}\left[e^{\frac{-t}{\tau_e}} - e^{\frac{-t}{\tau}}\right] = -20\left[e^{\frac{-t}{10m}} - e^{\frac{-t}{20m}}\right]$$

$\tau = \tau_e$:

$$\frac{V}{\tau_e} t e^{\frac{-t}{\tau_e}} = 10^3 t e^{\frac{-t}{10m}}$$



Section 7

Differentiator

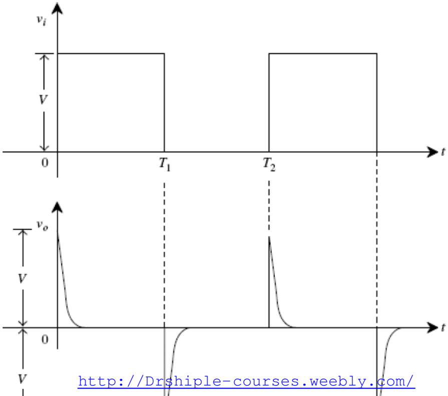
Differentiator

$$v_i = \frac{1}{C}\int idt + V_o \quad V_o = iR$$

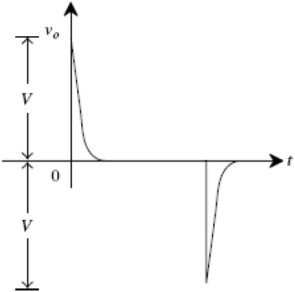
$$v_i = \frac{1}{RC}\int V_o dt + V_o$$

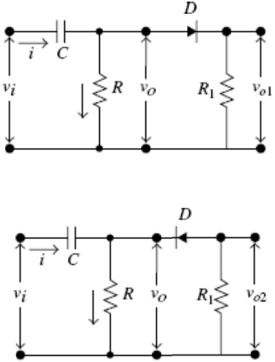
assume V_o is very small

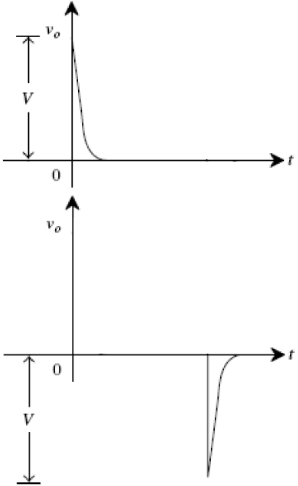
$$v_i = \frac{1}{RC}\int V_o dt \Rightarrow V_o = \tau\frac{dV_i}{dt}$$



Positive Spikes Vs. Negative Spikes







Limitation of RC Differentiator

$$\theta = tan^{-1}\frac{X_c}{R} = \frac{1}{\omega RC} = \frac{\omega_c}{\omega}$$

- When $\theta = 90^\circ$, the sine function at the input becomes a cosine function at the output, as is required in a differentiator.
- When $\omega_c/\omega = 100, \theta = 89.4^\circ$ which is nearly equal to 90°.
- A high-pass circuit behaves as a good differentiator only when $RC << T$.

Differentiator Vs. Integerator

Integrators are almost preferred over differentiators in analog computer applications for the following reasons:

1. It is easier to stabilize an integrator than a differentiator because the gain of an integrator decreases with frequency whereas the gain of a differentiator increases with frequency.
2. An integrator is less sensitive to noise voltages than a differentiator because of its limited bandwidth.
3. The amplifier of a differentiator may overload if the input waveform changes very rapidly.
4. It is more convenient to introduce initial conditions in an amplifier.