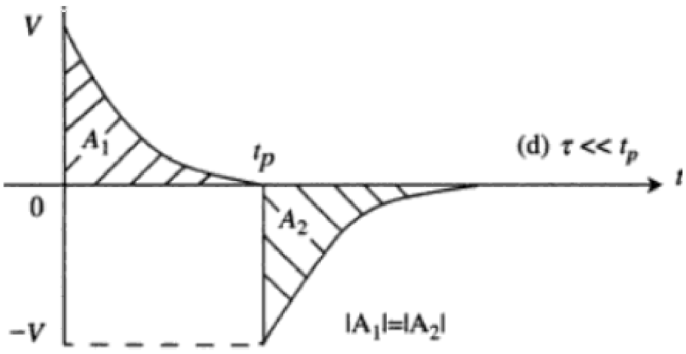
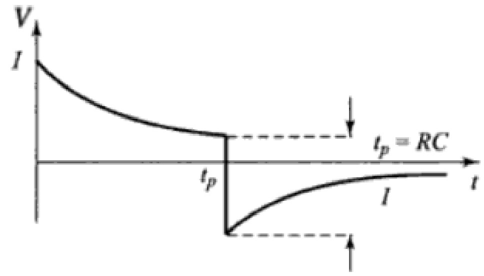
ANSWER THE FOLLOWING QUESTIONS:

1. Prove the DC level pulse response of v_o of the RC HPF is zero. In other words , the area above the reference (A1) equals the area under the reference (A2) as shown in the next figure[10 marks] [D$_c$]





Solution: For area A1:

$$v_o = Ve^{-\frac{t}{\tau}} \Rightarrow A1 = \int_0^{t_p} Ve^{-\frac{t}{\tau}}dt = V\int_0^{t_p} e^{-\frac{t}{\tau}}dt$$

$$\text{assume} :u = -\frac{t}{\tau} \Rightarrow du = -\frac{1}{\tau}dt \Rightarrow dt = -\tau du$$

$$A1 = -V\tau\int_0^{t_p} e^u du = -V\tau e^u\Big|_0^{t_p} = -V\tau e^{-\frac{t}{\tau}}\Big|_0^{t_p} = -V\tau\left[e^{-\frac{t_p}{\tau}} - 1\right]$$

$$A2 = V\left(e^{-\frac{t_p}{\tau}} - 1\right)\int_{t_p}^{\infty} e^{-\frac{t-t_p}{\tau}} = V\left(e^{-\frac{t_p}{\tau}} - 1\right)\left[-\tau e^{-\frac{\infty - t_p}{\tau}} + \tau e^{-\frac{t_p - t_p}{\tau}}\right]$$

$$= V\tau\left(e^{-\frac{t_p}{\tau}} - 1\right)$$

$$\therefore |A1| = |A2|$$

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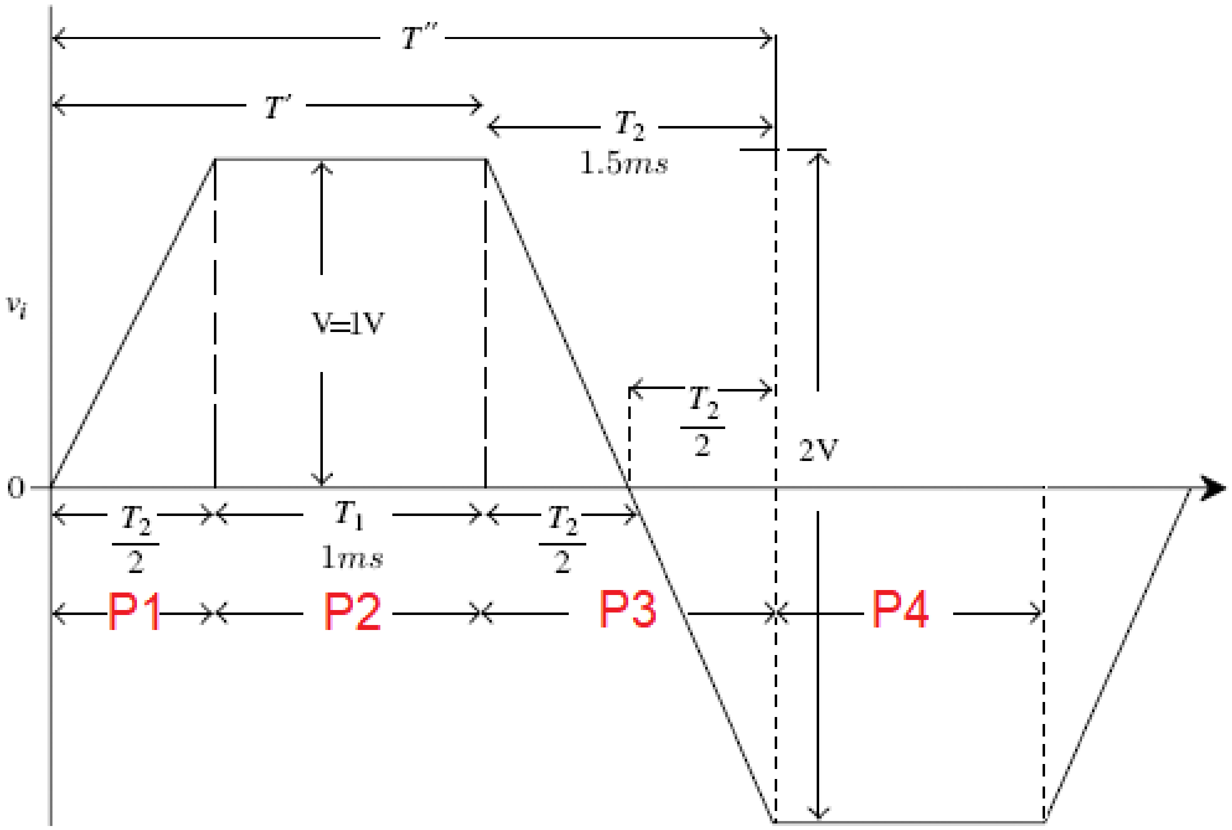
2. The input to a high-pass RC circuit in next figure is periodic and trapezoidal as indicated in next figure. Given that $T_1 = 1ms$ and $T_2 = 1.5ms$ and $\tau = 10ms$, find and sketch the steady-state output.. [10 marks] [D$_c$]



Solution:
The expected output is shown in the next figure:-
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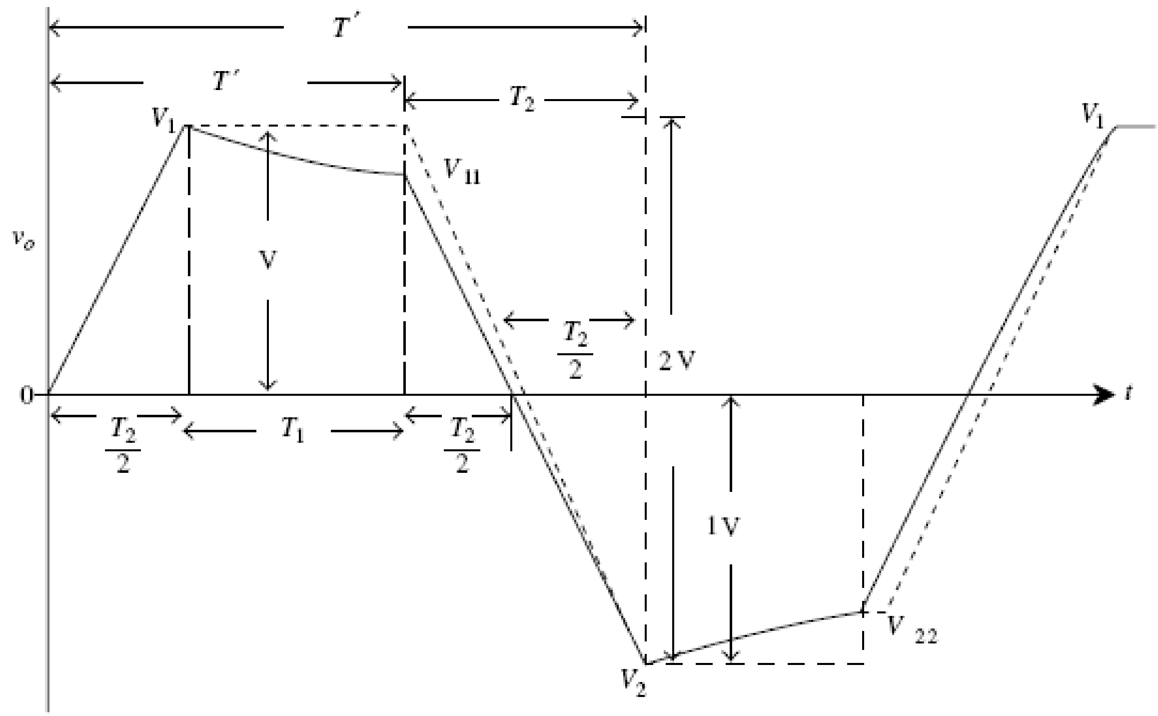
For period (P1):- (ramp section)

$$\tau = 10ms > 0.75m \qquad \therefore RC >> T \Rightarrow v_o(t) = \alpha t(1 - \frac{t}{2(RC)})$$

$$v_o(t) = \alpha\tau(1 - \frac{t}{2(RC)}) = \frac{2V}{T_2}t(1 - \frac{t}{20m})$$ output follows input

$$v_o(t)|_{T=\frac{T_2}{2}} = \frac{2V}{T_2}\frac{T_2}{2}(1 - \frac{T_2}{40}) \approx 1 = V_1$$



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For period (P2):- (Constant section)

$$v_o(t) = Ve^{-\frac{t}{\tau}} = V_1 e^{-100t}$$
$$V_{11} = v_o(t)|_{t=T_1} = V_1 e^{-100\times 1m} = 0.905V$$

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For period (P3):- (Inverse Ramp section)

$$\tau = 10ms > 1.5m \qquad \therefore RC >> T \Rightarrow v_o(t) = \alpha t(1 - \frac{t}{2(RC)})$$

$$v_o(t) = V_{11} - \left[\alpha t(1 - \frac{t}{2(RC)})\right] = V_{11} - \left[\frac{-2V}{T_2}T_2(1 - \frac{T_2}{20m})\right] = 0.905 - 1.85V \quad \text{output follows input}$$

$$V_2 = -0.945V$$

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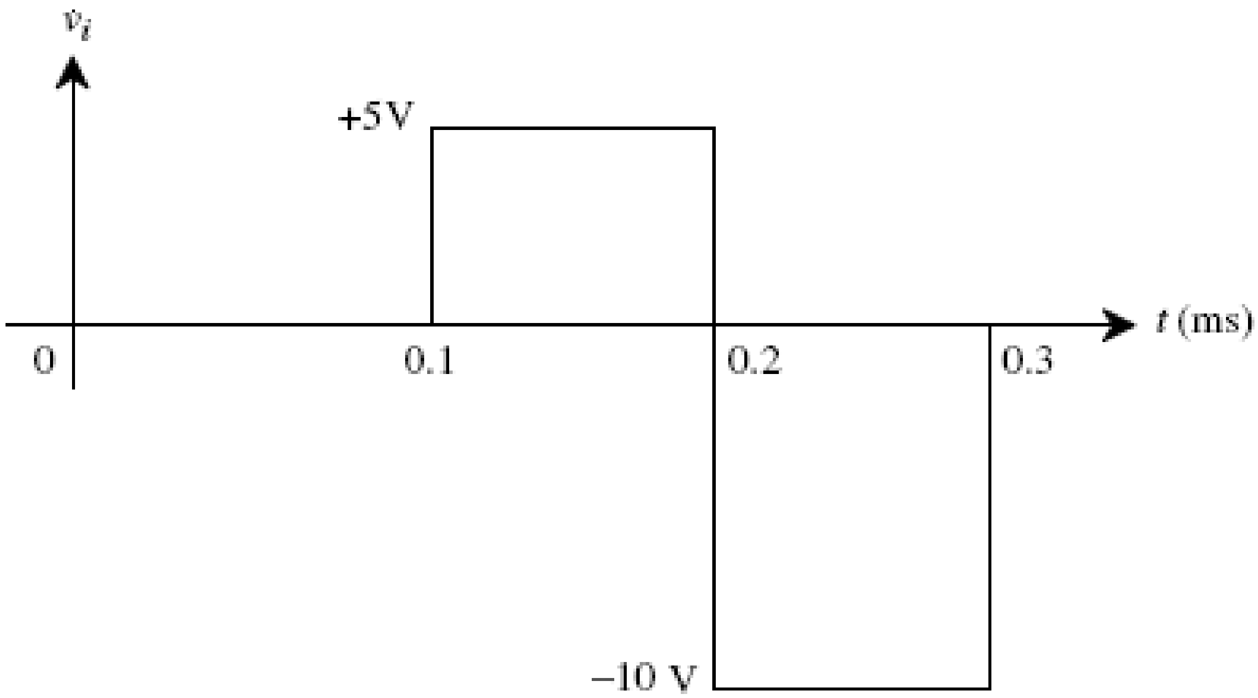
For period (P2):- (Constant section)

$$v_o(t) = Ve^{-\frac{t}{\tau}} = V_2 e^{-100t}$$
$$V_{22} = v_o(t)|_{t=T_1} = V_2 e^{-100\times 1m} = -0.855V$$

3. The input signal shown in next figure is applied to a RC high-pass circuit, whose time constant is 0.4 ms. Draw the output waveform and mark all voltages, assuming that the capacitor is initially uncharged. [10 marks] [D$_c$]



Solution:
The expected output is shown in the next figure:-
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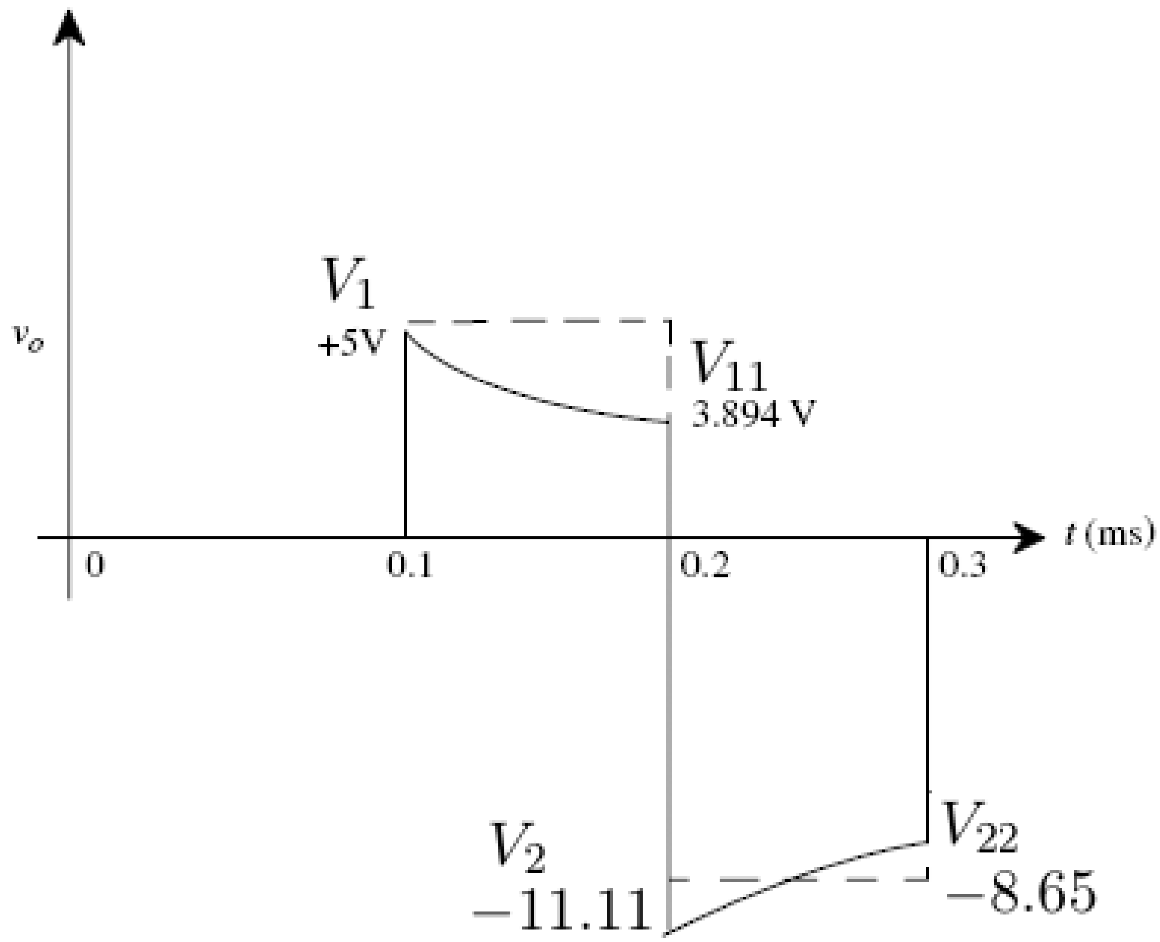
For period (P1):- (0.1 to 0.2 ms)

$$v_o(t) = Ve^{\frac{-t}{(RC)}} = 5e^{\frac{-t}{(0.4m)}}$$
$$v_o(t)|_{T=0.2} = 5e^{\frac{-0.1m}{(0.4m)}} = 3.89V$$



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For period (P2):- (at 0.2 ms theres a drop =-15 V)

$$V_2 = V_{11} - Drop\,voltage = 3.89 - 15 = -11.11V$$

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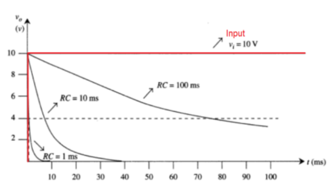
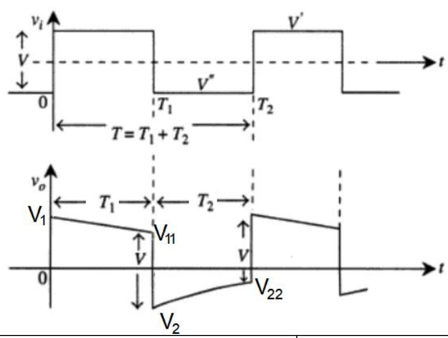
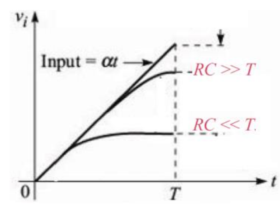
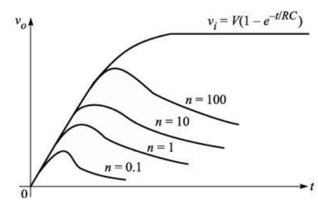
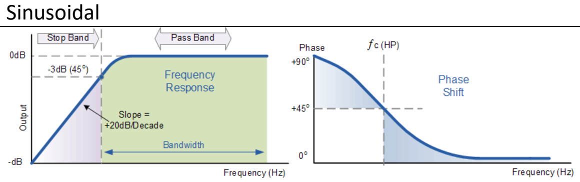
For period (P3):- (0.2 to 0.3)

$$v_o(t) = Ve^{\frac{-t}{(RC)}} = -11.11e^{\frac{-t}{(0.4m)}}$$
$$v_o(t)|_{T=0.3} = -11.11e^{\frac{-0.1m}{(0.4m)}} = -8.65V$$

Response of RC High Pass Filter

	Step	Pulse	Square	Ramp	Exponential
Output signal					
V_o	$Ve^{-\frac{t}{\tau}}$	$v_o = \begin{cases} Ve^{-\frac{t}{\tau}} & 0 < t < t_p \\ V\left(e^{-\frac{t_p}{\tau}} - 1\right) e^{-\frac{t-t_p}{\tau}} & t > t_p \end{cases}$ $V_2 = V_{11} - V$	$T_1 = T_2.$ $V_1 = -V_2$ $V_{11} = -V_{22}$ $V_1 = \dfrac{V}{(1 + e^{-\frac{T}{2\tau}})}$	$\begin{cases} RC \approx T & \alpha RC(1 - e^{-\frac{t}{RC}}) \\ RC << T & \alpha RC \\ RC >> T & \alpha t(1 - \frac{t}{2(RC)}) \end{cases}$	$\begin{cases} \tau \neq \tau_e, & \frac{V}{(\frac{\tau_e}{\tau} - 1)} \left[e^{\frac{-t}{\tau_e}} - e^{\frac{-t}{\tau}}\right] \\ \tau >> \tau_e, & \frac{-V}{(\frac{\tau_e}{\tau} - 1)} \left[e^{\frac{-t}{\tau}}\right] \\ \tau = \tau_e, & \frac{V/\tau_e}{\left(S + \frac{1}{\tau_e}\right)^2} = \frac{V}{\tau_e} t e^{\frac{-t}{\tau_e}} \end{cases}$

	Sinusoidal
Output signal	
V_o	$\lvert T(S) \rvert = \dfrac{\frac{\omega}{\omega_c}}{\sqrt{1 + (\frac{\omega}{\omega_c})^2}}$ $\theta(\omega) = 90 - \tan^{-1}(\dfrac{\omega}{\omega_c})$

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