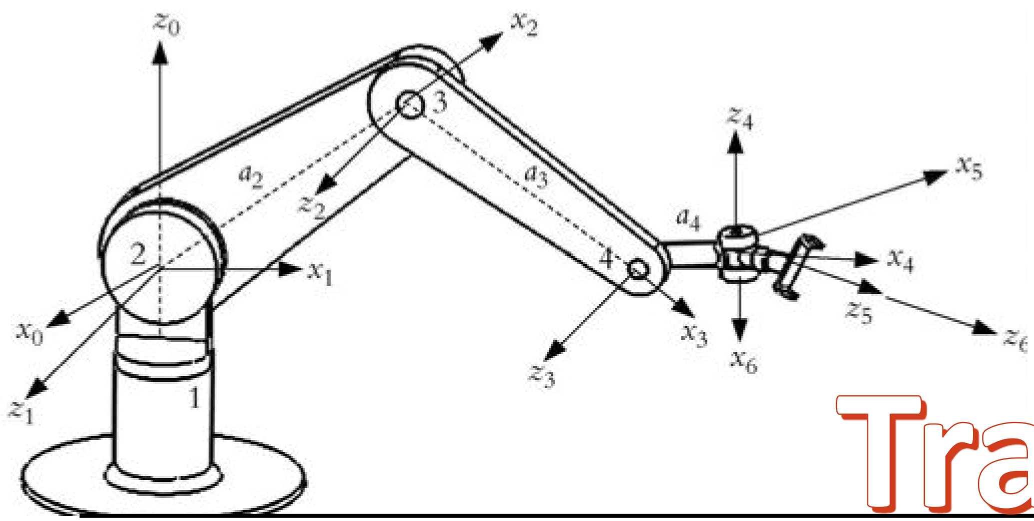
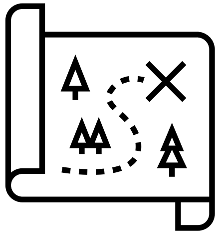


Trajectory Planning

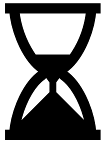
By: Dr. Mustafa Shiple

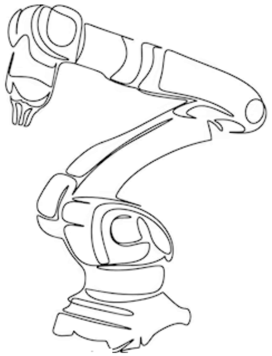
The challenges of online presentations

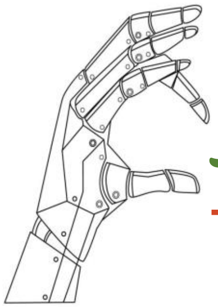
Path and trajectory planning relates to the way a robot is moved from one location to another in a controlled manner



- **Path** : A is defined as the collection of a sequence of locations (configurations a robot makes) to go from one place to another without regard to the timing of these configurations.
- **Trajectory** is the collection of the locations (configurations) with respect to *time*.

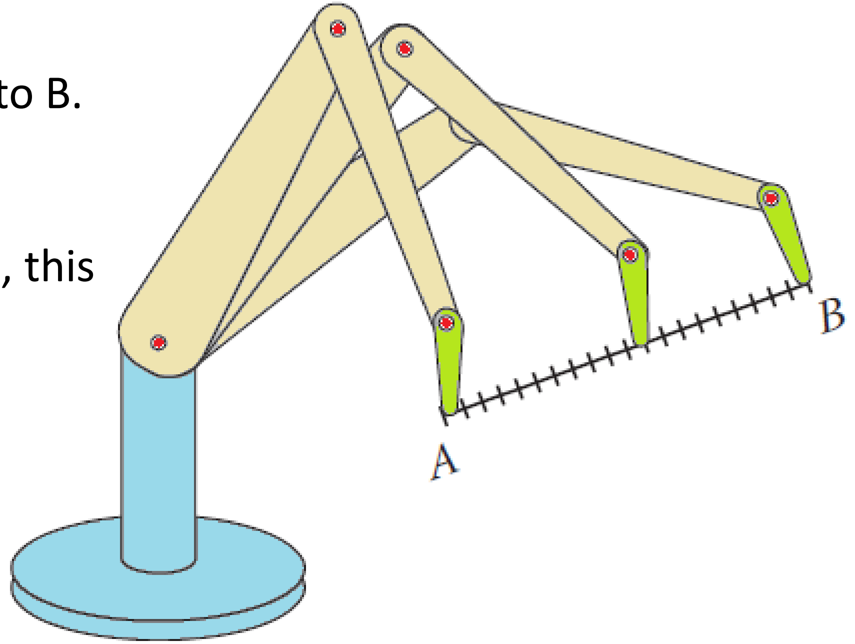




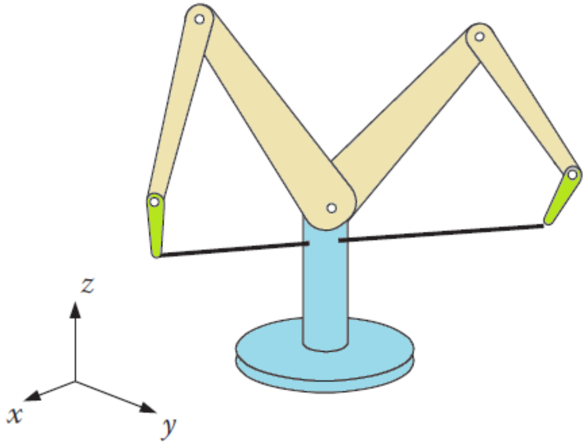


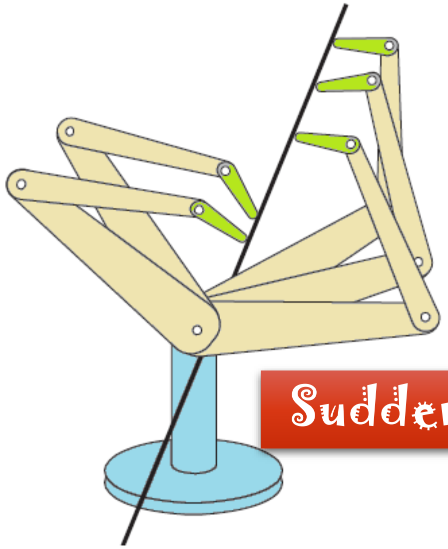
Joint-Space vs. Cartesian-Space Descriptions

- The description of the motion to be made by the robot with its joint values is called the ***joint-space description.***
- ***Inverse kinematics ,*** is not used to transfer end effector from point A to B.
- ***Increasing intermediate*** points between A and B , increasing path definitions.
- When robot course is impossible and yields an unsatisfactory solution, this is called ***singularities***.



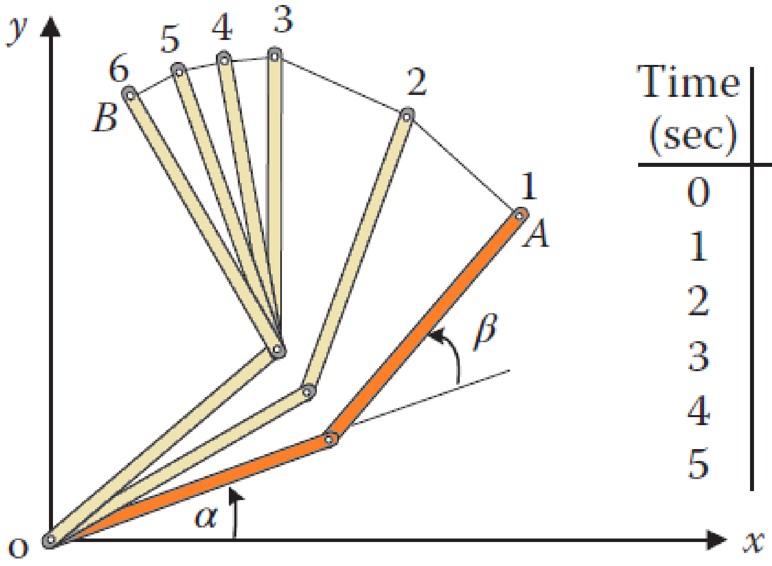
Obstacles





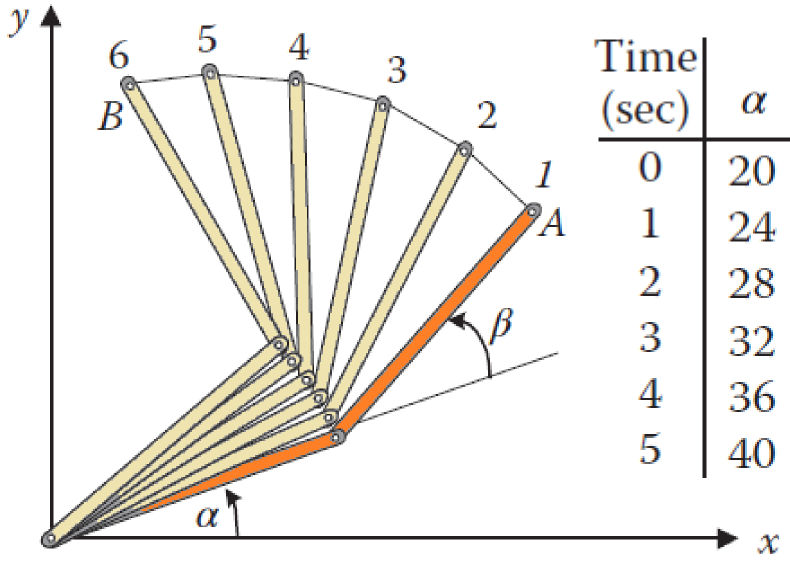
Sudden change!!

Trajectory Planning (undefined)

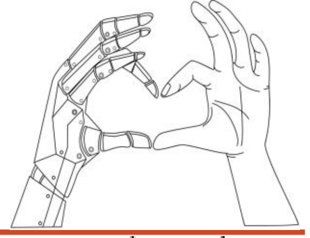


Time (sec)	α	β
0	20	30
1	30	40
2	40	50
3	40	60
4	40	70
5	40	80

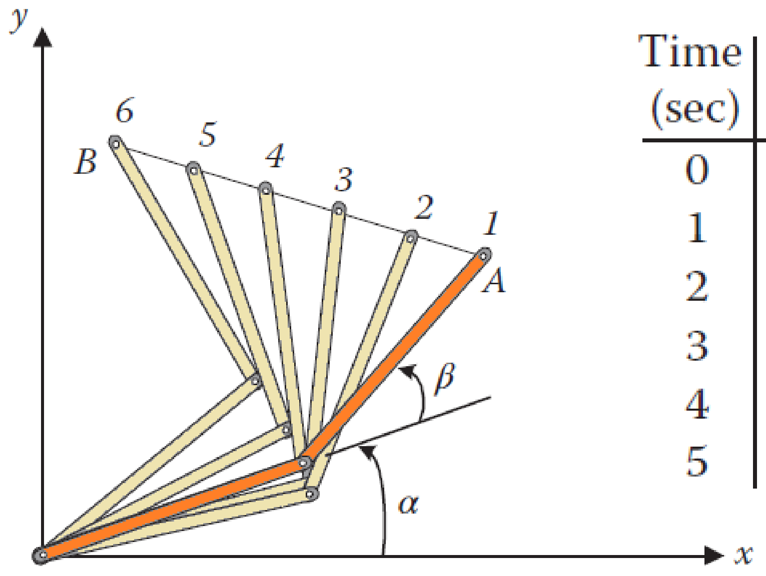
Keep going for smooth moving



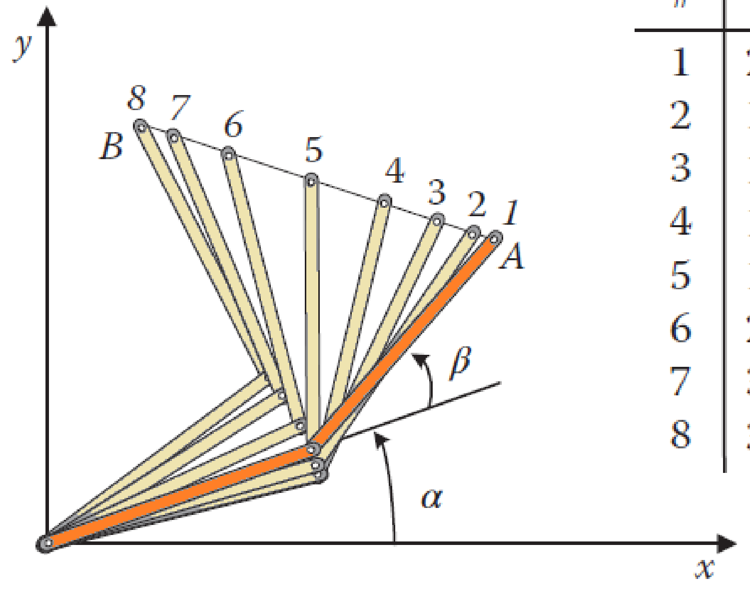
Time (sec)	α	β
0	20	30
1	24	40
2	28	50
3	32	60
4	36	70
5	40	80



Trajectory Planning (pre-defined)



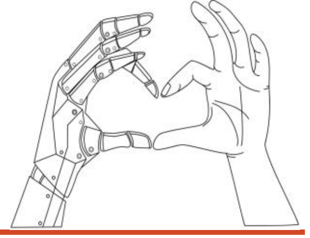
Time (sec)	α	β
0	20	30
1	13	56
2	15	69
3	19	78
4	28	82
5	40	80



#	α	β
1	20	30
2	16	40
3	15	50
4	13	59
5	15	70
6	25	80
7	32	81
8	37	80

- ✓ Define predefined path.
- Violate predefined velocity 10º/sec.
- α *angle* $+/-$
- β changes irregularly vs same time (irregular speed (discrete))

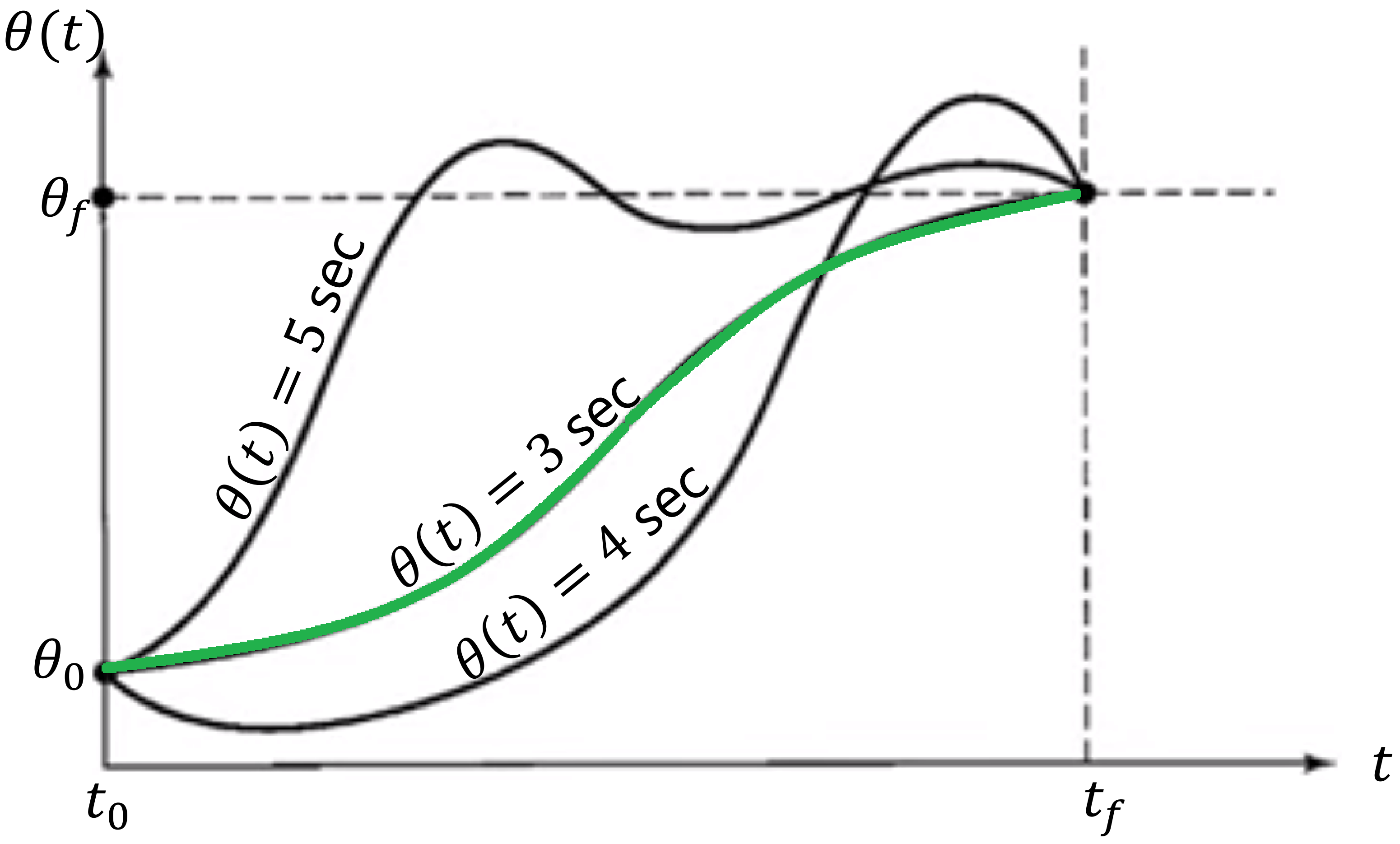
Joint-Space Trajectory



A motor should be able to provide the **accelerations**[1] and **velocities**[2] needed to control joint movements.

1. Third-Order Polynomial Trajectory Planning.
2. Fifth-Order Polynomial Trajectory Planning.
3. Linear Segments with Parabolic Blends.

Third-Order Polynomial Trajectory Planning

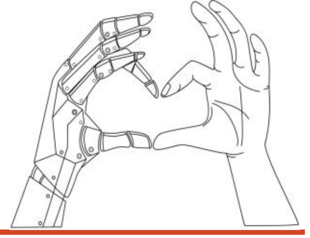


$\theta(t)$ Joint angle varied by time
initial analge

$\theta(t_0) = \theta_0$ initial angle
$\theta(t_f) = \theta_f$ final angle
$\dot{\theta}(t_0) = 0$ initial velocity
$\dot{\theta}(t_f) = 0$ final velocity

$$\theta(t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3$$

Third-Order Polynomial Trajectory Planning



$$\theta(t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3$$

$$\dot{\theta}(t) = c_1 + 2c_2 t + 3c_3 t^2$$

$$\theta(t)|_{t=0} = c_0 + c_1 t + c_2 t^2 + c_3 t^3 = c_0 = \theta_0$$

$$\dot{\theta}(t)|_{t=0} = c_1 + 2c_2 t + 3c_3 t^2 = c_1 = 0$$

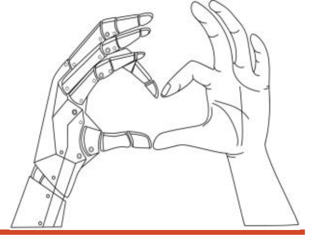
$\theta(t_0) = \theta_0$ initial angle

$\theta(t_f) = \theta_f$ final angle

$\dot{\theta}(t_0) = 0$ initial velocity

$\dot{\theta}(t_f) = 0$ final velocity

Third-Order Polynomial Trajectory Planning (example)



We intend to have the first joint of a 6-axis robot go from an initial angle of 30º to a final angle of 75º in 5 seconds. Using a third-order polynomial, calculate the joint angle at 1, 2, 3, and 4 seconds.

$$\theta(t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3$$

$$\theta(t)|_{t=0} = c_0 = 30^\circ$$

$$\theta(t)|_{t=5} = (30) + 5c_1 + 25c_2 + 75c_3 = 75^\circ$$

$$\Rightarrow 5c_2 + 15c_3 = 9 \quad (1)$$

$$\dot{\theta}(t) = c_1 + 2c_2 t + 3c_3 t^2$$

$$\dot{\theta}(t)|_{t=0} = c_1 = 0$$

$$\dot{\theta}(t)|_{t=5} = (0) + 10c_2 + 75c_3 = 0$$

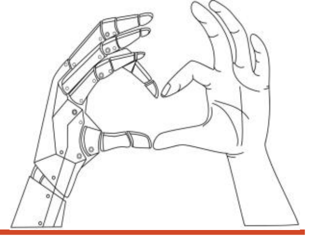
$$\Rightarrow c_2 + 7.5c_3 = 0 \quad (2)$$

$$\begin{cases} c_0 = 30 \\ c_1 = 0 \\ c_2 = 5.4 \\ c_3 = -0.72 \end{cases}$$

$$\theta(t) = 30 + 5.4t^2 - 0.72\,t^3$$

$$\dot{\theta}(t) = 10.8\,t - 2.16\,t^2$$

Third-Order Polynomial Trajectory Planning (example)



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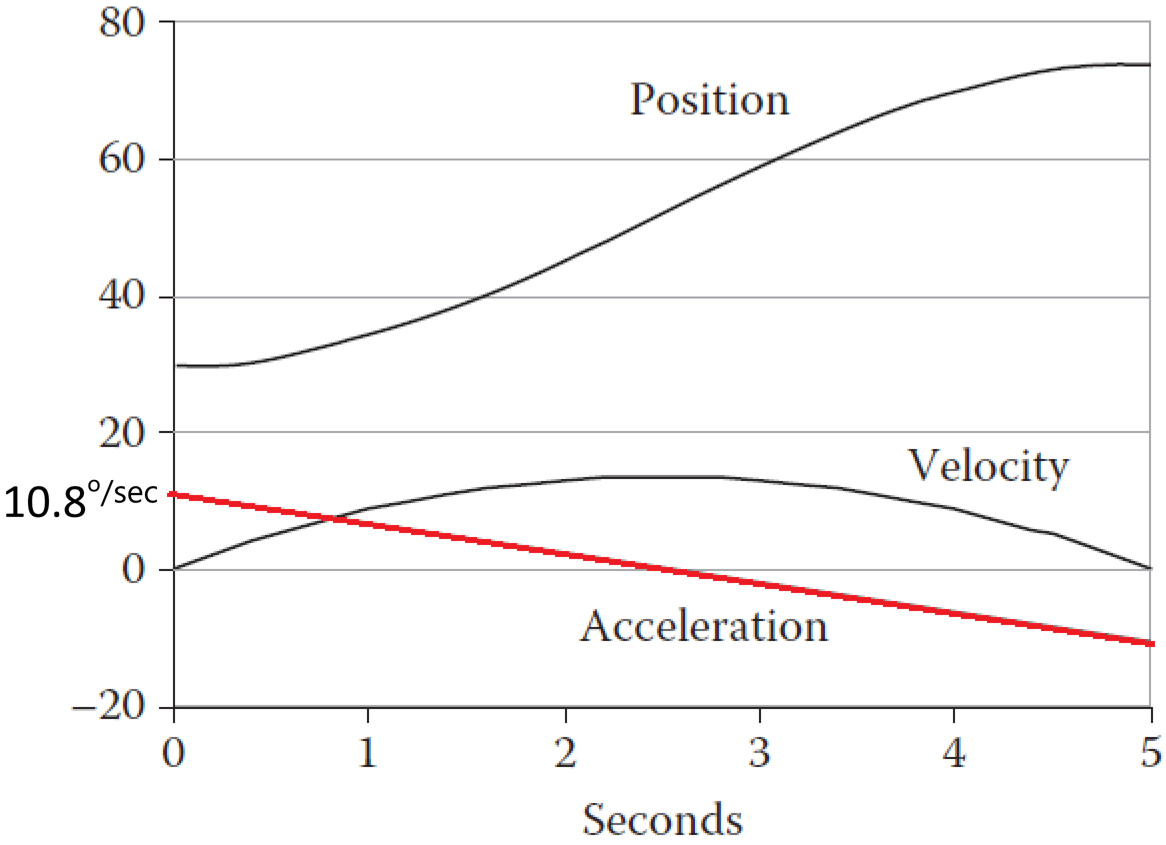
$$\theta(t) = 30 + 5.4t^2 - 0.72\,t^3$$

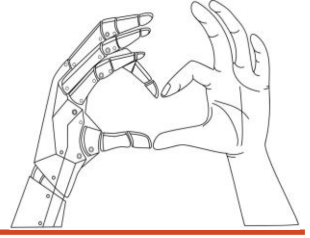
$$\dot{\theta}(t) = 10.8\,t - 2.16\,t^2$$

$$\theta(1) = 34.68^\circ,\ \theta(2) = 45.84^\circ,\ \theta(3) = 59.16^\circ,\ \theta(4) = 70.32^\circ$$

$$\ddot{\theta}(t) = 10.8 - 4.32\,t$$



Third-Order Polynomial Trajectory Planning (example)



continue to the next point, where the joint is to reach 105∘ in another 3 seconds. Draw the position, velocity, and acceleration curves for the motion.

$$\theta(t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3$$

$$\theta(t)|_{t=0} = c_0 = 75^\circ$$
$$\theta(t)|_{t=3} = (75) + 3c_1 + 9c_2 + 27c_3 = 105^\circ$$

$$\Rightarrow 3c_2 + 9c_3 = 10 \quad (1)$$

$$\dot{\theta}(t) = c_1 + 2c_2 t + 3c_3 t^2$$

$$\dot{\theta}(t)|_{t=0} = c_1 = 0$$
$$\dot{\theta}(t)|_{t=3} = (0) + 6c_2 + 27c_3 = 0$$

$$\Rightarrow 6c_2 + 27c_3 = 0 \quad (2)$$

$$\begin{cases} c_0 = 75 \\ c_1 = 0 \\ c_2 = 10 \\ c_3 = -2.22 \end{cases}$$

$$\theta(t) = 75 + 10t^2 - 2.22\, t^3$$

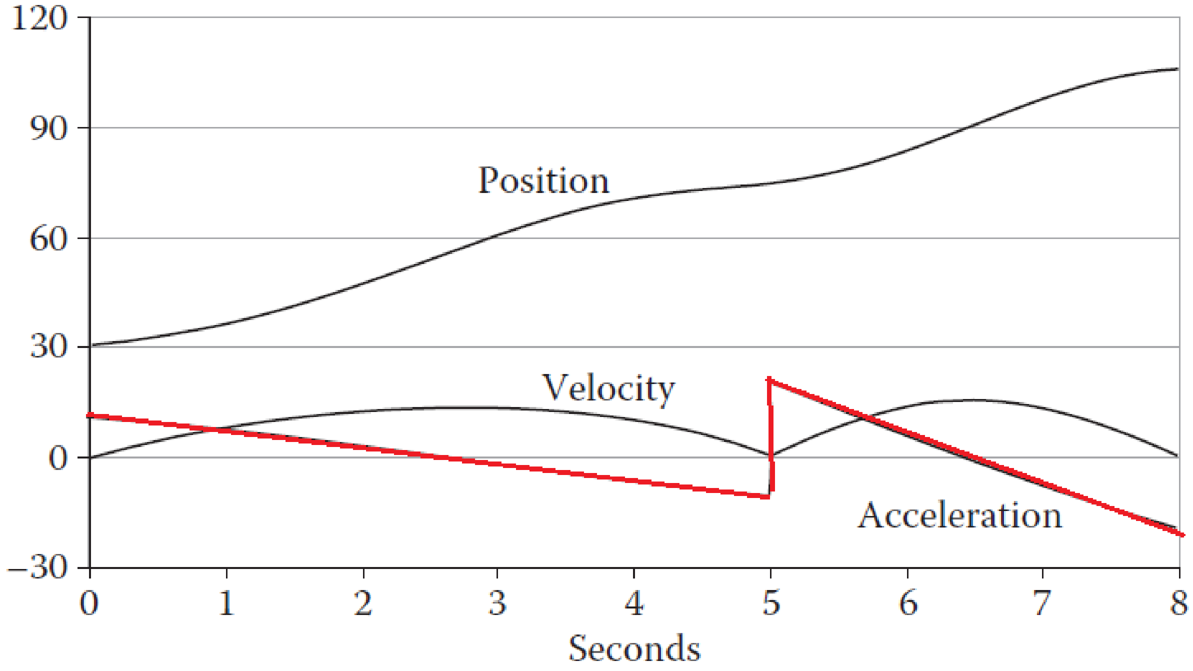
$$\dot{\theta}(t) = 20\, t - 6.66\, t^2$$

Third-Order Polynomial Trajectory Planning (example)

$$\theta(t) = 75 + 10t^2 - 2.22\, t^3$$

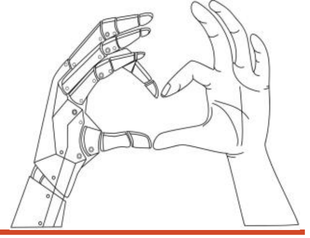
$$\dot{\theta}(t) = 20\, t - 6.66\, t^2$$

$$\ddot{\theta}(t) = 20 - 13.33\, t$$



Instantaneous change in acceleration (jerk).

Vibration



Vibration-Minimizing Motion Retargeting for Robotic Characters

Shayan Hoshyari Hongyi Xu Espen Knoop

Stelian Coros Moritz Bächer

Disney Research ETH zürich THE UNIVERSITY OF BRITISH COLUMBIA

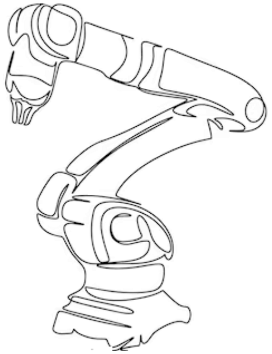
<this video has audio>

Fifth-Order Polynomial Trajectory Planning

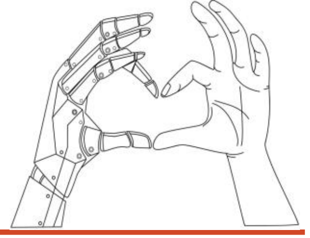
$$\theta(t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3 + c_4 t^4 + c_5 t^5$$

$$\dot{\theta}(t) = c_1 + 2c_2 t + 3c_3 t^2 + 4c_4 t^3 + 5c_5 t^4$$

$$\ddot{\theta}(t) = 2c_2 + 6c_3 t + 12c_4 t^2 + 20c_5 t^3$$



Fifth-Order Polynomial Trajectory Planning (example)



We intend to have the first joint of a 6-axis robot go from an initial angle of 30º to a final angle of 75º in 5 seconds. Using a third-order polynomial, calculate the joint angle at 1, 2, 3, and 4 seconds.

$$\theta(t) = c_0 + c_1 t + c_2 t^2 + c_3 t^3 + c_4 t^4 + c_5 t^5$$

$$\dot{\theta}(t) = c_1 + 2c_2 t + 3c_3 t^2 + 4c_4 t^3 + 5c_5 t^4$$

$$\ddot{\theta}(t) = 2c_2 + 6c_3 t + 12c_4 t^2 + 20c_5 t^3$$

$\theta(0) = 30º$, $\dot{\theta}(0) = 0º$/sec , $\ddot{\theta}(0) = +5$ º/sec²

At time = 0 sec

$$\theta(0) = c_0 = 30$$

$$\dot{\theta}(0) = c_1 = 0$$

$$\ddot{\theta}(0) = 2c_2 = +5 => c_2 = 2.5$$

$\theta(5) = 75º$, $\dot{\theta}(5) = 0º$/sec , $\ddot{\theta}(5) = -5$ º/sec²

$$\theta(5) = 62.5 + 125c_3 + 625c_4 + 3125c_5 = 75$$

$$\dot{\theta}(t) = 25c_2 + 75c_3 t^2 + 500c_4 + 3125c_5 = 0$$

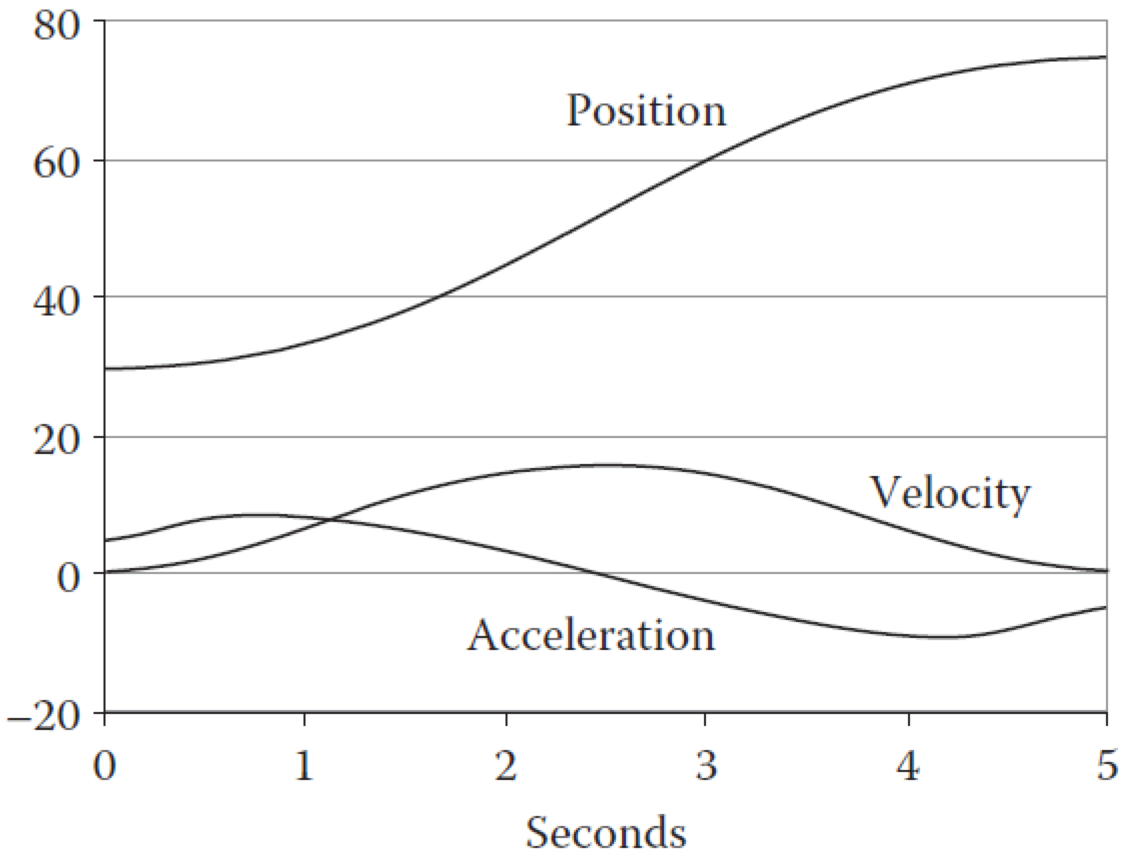
$$\ddot{\theta}(t) = 5 + 30c_3 + 300c_4 + 2500c_5 = -5$$

Fifth-Order Polynomial Trajectory Planning (example)

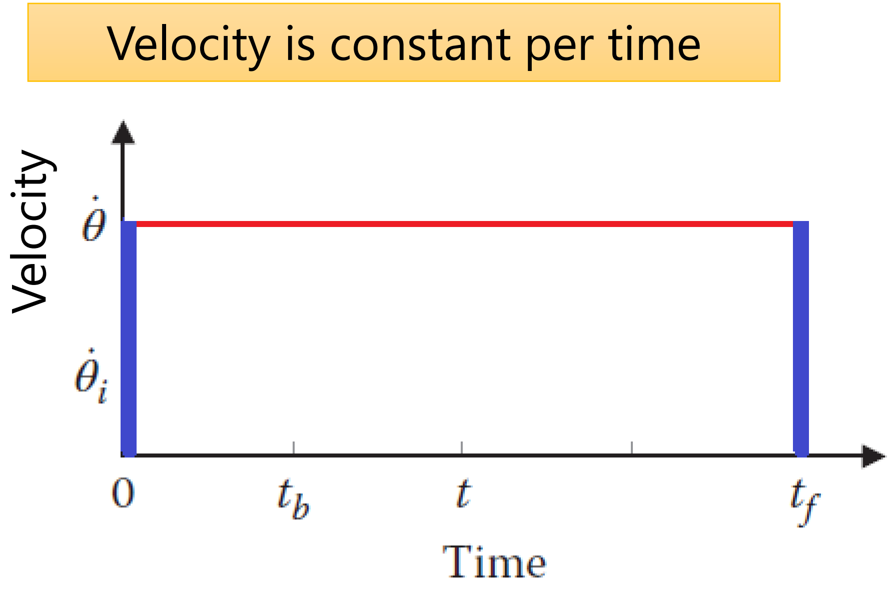
We intend to have the first joint of a 6-axis robot go from an initial angle of 30º to a final angle of 75º in 5 seconds. Using a third-order polynomial, calculate the joint angle at 1, 2, 3, and 4 seconds.

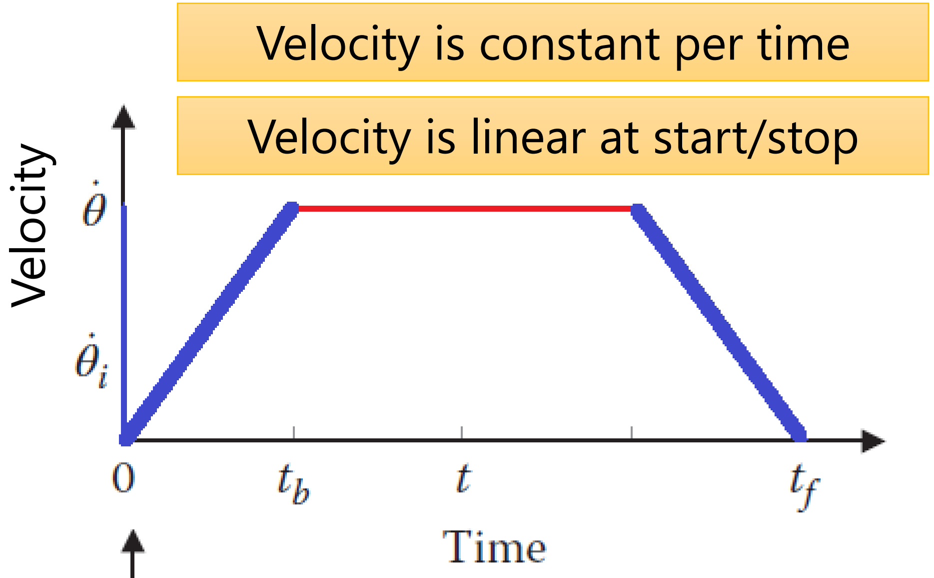
$$\begin{cases} c_0 = 30 \\ c_1 = 0 \\ c_2 = 2.5 \\ c_3 = 1.6 \\ c_4 = -0.58 \\ c_5 = 0.0464 \end{cases}$$

Velocity is varied per time



Linear Segments with Parabolic Blends

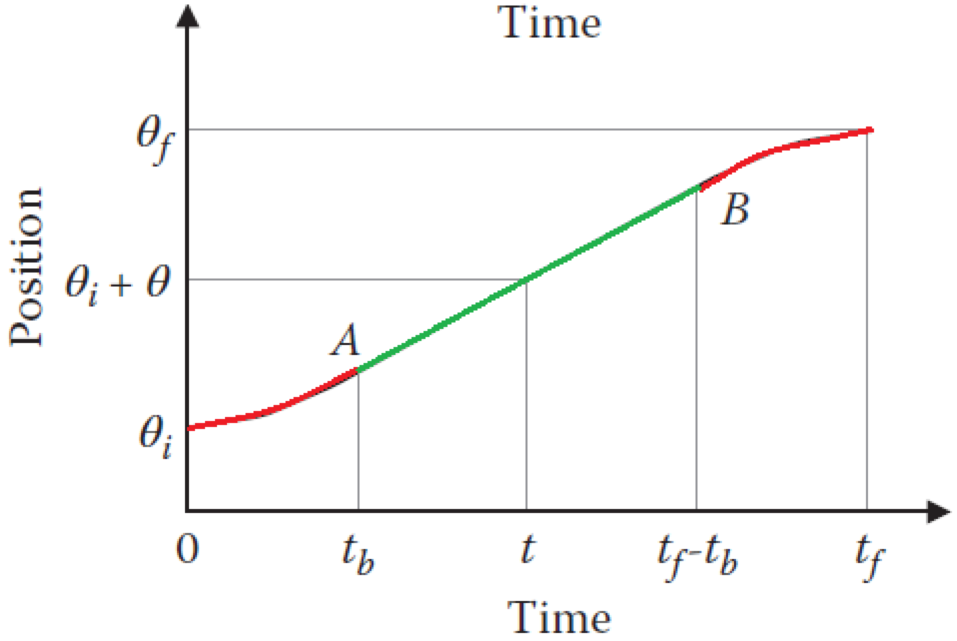


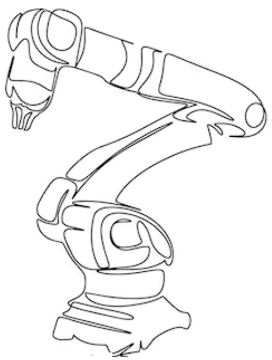


$$\theta(t) = c_0 + c_1 t + 0.5 c_2 t^2$$

$$\dot{\theta}(t) = c_1 + c_2 t$$

$$\ddot{\theta}(t) = c_2$$

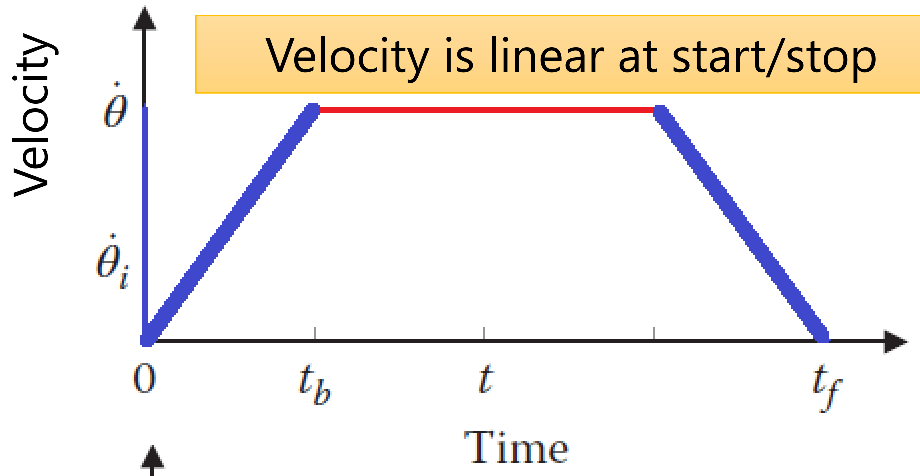


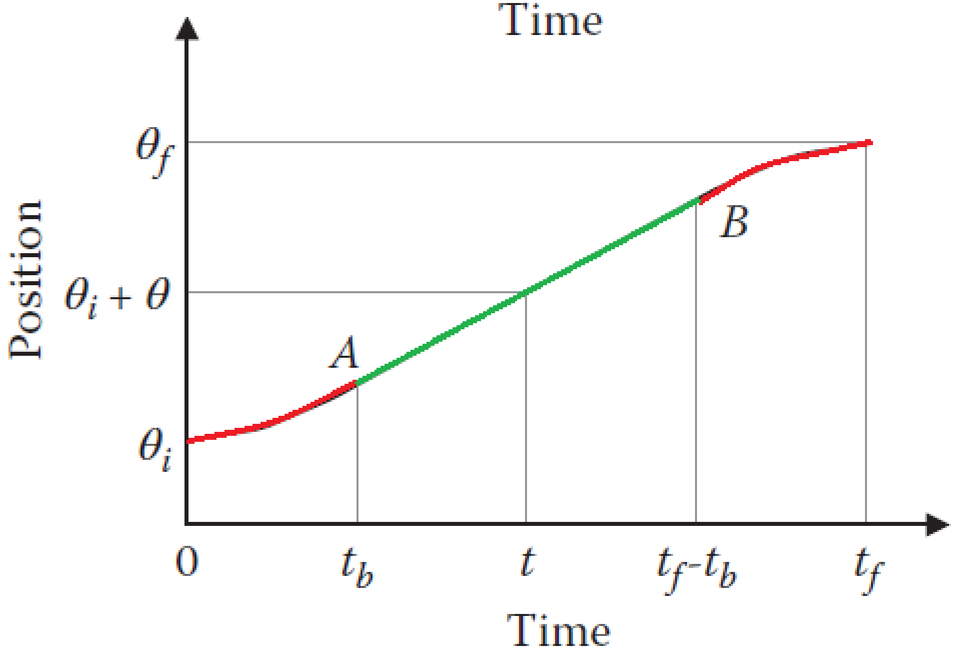


Linear Segments with Parabolic Blends

Velocity is constant per time

Velocity is linear at start/stop





$$\theta(t) = c_0 + c_1 t + 0.5 c_2 t^2$$

$$\dot{\theta}(t) = c_1 + c_2 t$$

$$\ddot{\theta}(t) = c_2$$

At soft beginning:-

$$\theta(0) = c_0 = \theta_i$$

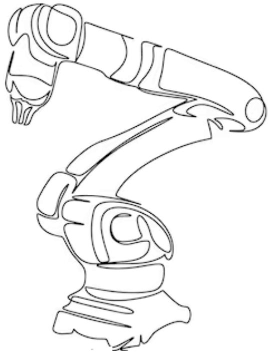
$$\dot{\theta}(0) = c_1 = 0$$

$$\ddot{\theta}(0) = c_2 = \ddot{\theta}$$

$$\theta(t) = \theta_i + 0.5 c_2 t^2$$

$$\dot{\theta}(t) = c_2 t$$

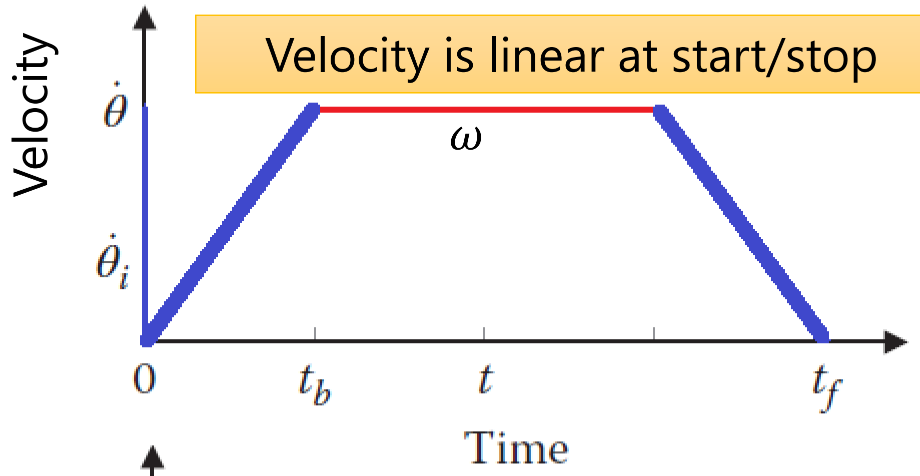
$$\ddot{\theta}(t) = c_2$$

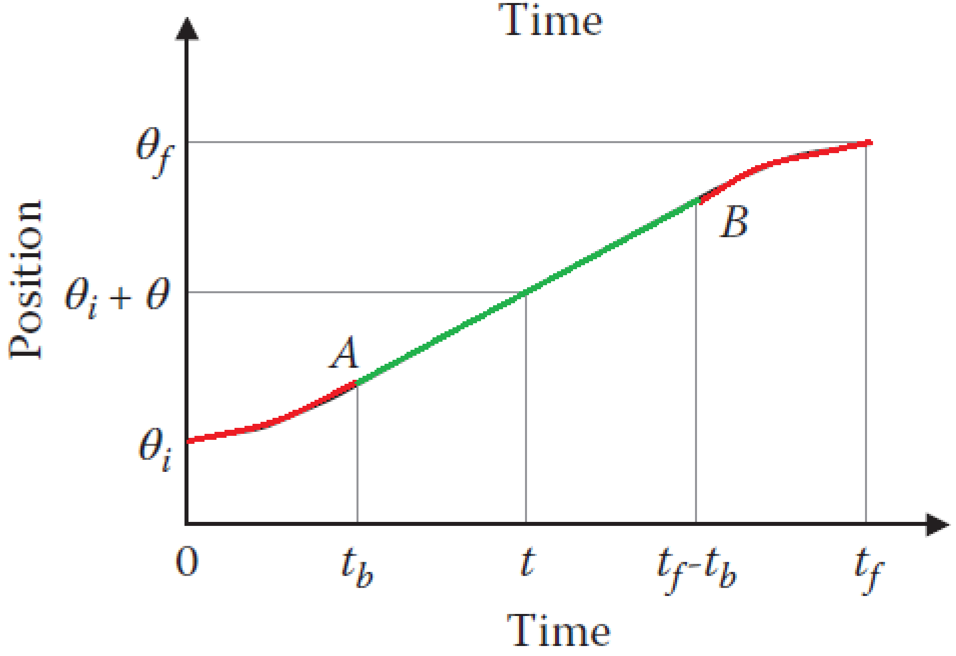


Linear Segments with Parabolic Blends

Velocity is constant per time

Velocity is linear at start/stop





At linear segment:-

$$\theta(A) = \theta_0 + 0.5c_2 t_b^2$$

$$\dot{\theta}(A) = c_2 t = \omega$$

$$\ddot{\theta}(t) = 0$$

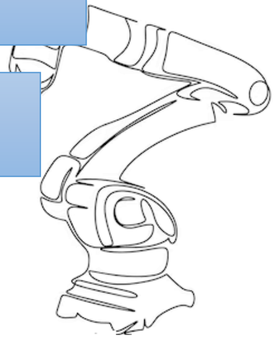
$$\theta(B) = \theta_A + \omega\left(\left(t_f - t_b\right) - t_b\right)$$
$$\theta(B) = \theta_A + \omega\left(t_f - 2t_b\right)$$

$$\dot{\theta}(B) = c_2 t = \omega$$

$$\theta(f) = \theta_B + (\theta_A - \theta_i)$$

$$\dot{\theta}(f) = 0$$

$$t_b = \frac{\theta_i - \theta_f + \omega t_f}{\omega}$$



example

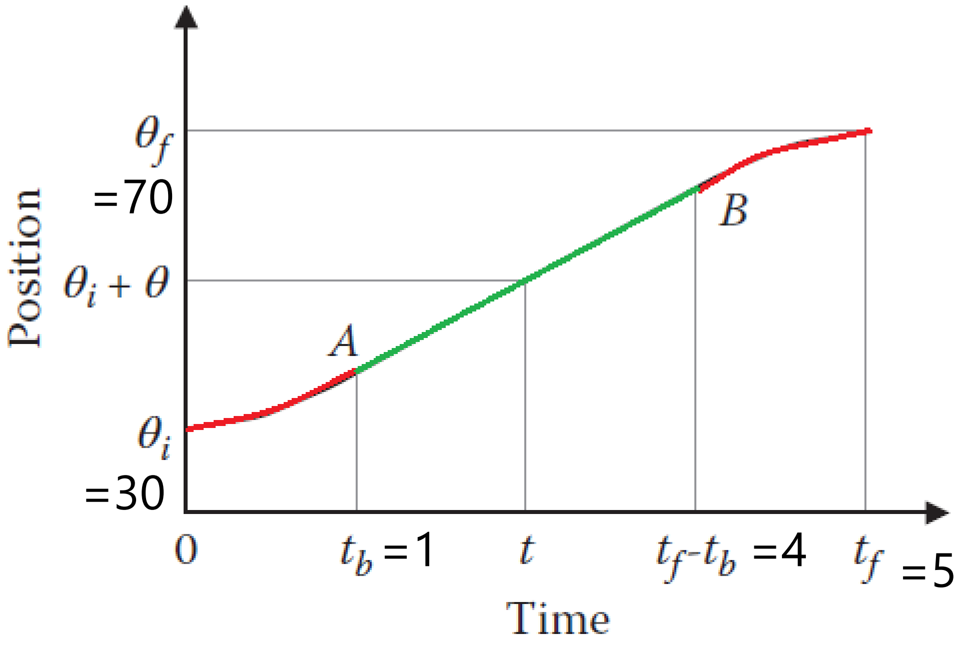
Joint 1 of the 6-axis robot is to go from an initial angle of 30º to the final angle of 70º in 5 seconds with a cruising velocity of $\omega 1$ =10 /sec. Find the necessary time for blending, and plot the joint positions, velocities, and accelerations.

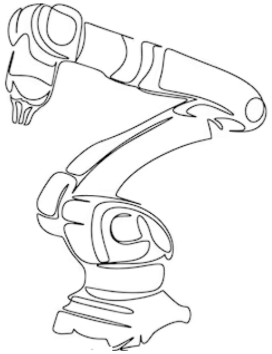
$$t_b = \frac{\theta_i - \theta_f + \omega t_f}{\omega} = \frac{30 - 70 + 10 \times 5}{10} = 1\,sec$$

$\theta(t) = \theta_i + 0.5c_2t^2$ $=30+5t^2$

$\dot{\theta}(t) = c_2t$ $=10t$

$\ddot{\theta}(t) = c_2$ $=10$





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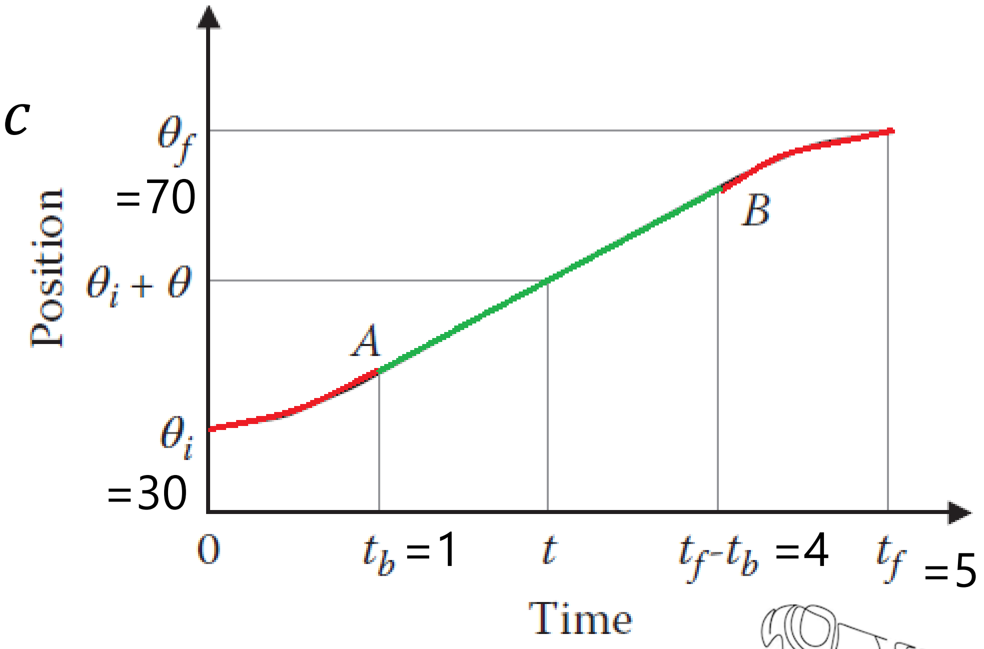
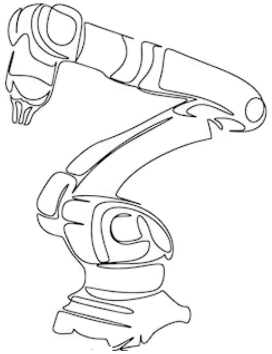
$$t_b = \frac{\theta_i - \theta_f + \omega t_f}{\omega} = \frac{30 - 70 + 10 \times 5}{10} = 1\ sec$$



$$\dot{\theta}(A) = c_2 t = 10$$

$$\theta(B) = \theta_A + \omega\left(t_f - 2t_b\right) = \theta_A + 10\left(t_f - 2\right)$$



Joint 1 of the 6-axis robot is to go from an initial angle of 30º to the final angle of 70º in 5 seconds with a cruising velocity of $\omega 1 = 10$ /sec. Find the necessary time for blending, and plot the joint positions, velocities, and accelerations.

$$t_b = \frac{\theta_i - \theta_f + \omega t_f}{\omega} = \frac{30 - 70 + 10 \times 5}{10} = 1\ sec$$

$$\rightarrow \begin{cases} \theta(t) = \theta_f - \dfrac{\omega}{2t_b}(t_f - t)^2 \\ \dot{\theta}(t) = \dfrac{\omega}{t_b}(t_f - t) \\ \ddot{\theta}(t) = -\dfrac{\omega}{t_b} \end{cases} \qquad \begin{cases} \theta = 70 - 5(5-t)^2 \\ \dot{\theta} = 10(5-t) \\ \ddot{\theta} = -10 \end{cases}$$